THE FOURTH FREDERIK J. PLANTEMA MEMORIAL LECTURE

FATIGUE LIFE OF STRUCTURAL COMPONENTS UNDER RANDOM LOADING

by

E. Gassner

Laboratorium für Betriebsfestigkeit,
Darmstadt, Germany (FR).

SUMMARY

There are many experimental methods available for evaluating the fatigue strength of engineering materials and for defining the fatigue life of structures subjected to random loading. The scale extends from constant amplitude tests to tests in which both the amplitudes and the frequencies occurring under service conditions are duplicated. Selection of a test method within the aforementioned range of application is guided by considerations as to whether or not service loading is realistically simulated or at least tolerably simplified. However, this raises problems which cannot be resolved, even with the most sophisticated testing equipment available, without making compromises with regard to test duration, cost and comparability of test results.

In the immediate past 'individual randomness' of test loading led to numerous results which were rather isolated and confusing, making understanding between research workers often difficult, or even impossible.

The aim of the present paper is to propose a 'standard' procedure based on extensive experience and on relevant literature which, it is expected, will contribute to an improvement of the state of knowledge in most areas concerned with fatigue life prediction.

LBF Report No.FB-107 (1973)

FOREWORD

It was through Frederik J. Plantema and the activities of the International Committee on Aeronautical Fatigue, founded by him in 1951, that the Federal Republic of Germany established connections with international research in the field of aeronautical fatigue so soon (1956) after the second world war.

I think you will agree with me when I say that the personality of Frederik Plantema was impressed on the character of this nonbureaucractic organization and that without his influence it would have been inconceivable that this team of scientists, who today are so closely linked, could have given such positive impulses to applied research in aircraft fatigue.

I first met Frederik Plantema at the IUTAM Colloquium on Fatigue in Stockholm in 1955. In the following years there arose a very close contact with Frederik Plantema which was not restricted only to ICAF activities. I often asked him for his advice on the fatigue strength problems of modern German aircraft structures and I always found a colleague who was readily accessible to provide help with my problems.

Therefore it is not only an honour for me to present the Fourth Plantema Memorial Lecture, but it is also an opportunity to make a contribution to a problem, the significance of which, in future aeronautical fatigue, had already been realized by Frederik Plantema ten years ago.

Dr. Plantema's life history and his contribution to aeronautical research

Frederik J. Plantema was born on 21 October 1911 at Leeuwarden in The Netherlands. He graduated from the Technological University of Delft at the age of 21 years. For a short period he was assistant to Professor Biezeno at the same University, and in 1934 he joined the National Luchtvaartlaboratorium (NLL) in Amsterdam. In 1945 he was charged with the leadership of the Structures Department and in 1950, when the Structures Department and the Materials Department became one joint department, he was appointed to be the head of it, which he remained until his death.

Plantema had been working on airworthiness problems, aircraft structures, stress analysis and related problems. A large number of papers and reports could easily illustrate the variety of subjects. He wrote a book in Dutch on the stress analysis of aircraft structures while shortly before his death his book "Sandwich Construction, The Bending and Buckling of Sandwich Beams, Plates and Shells" was published in the United States. Another field of his interest was concerned with loads on aircraft and the response of the structure to these loads. It was the integration of these issues and his general interest in airworthiness problems that may explain why he focussed so much effort on fatigue. In 1949 he had completed an extensive study of information available at that time regarding fatigue of structures and structural components. From then on the fatigue activities of the NLR were steadily increased and cumulative fatigue damage became one of the prominent subjects. In 1955 Plantema presented the first NLR paper on this subject at the IUTAM Colloquium on Fatigue, held at Stockholm.

In 1951 Plantema had taken the initiative for the foundation of the International Committee on Aeronautical Fatigue. A preliminary meeting then had taken place in September 1951 at Cranfield. From this day till his unexpected death in November 1966 he led the organization of ICAF as its Secretary.

CONTENTS

1. INTRODUCTION

Increasing demands for economy and safety in highly utilized structures under random loading necessitate a fatigue life to failure only slightly in excess of the service life. This presupposes reliable calculation and experimental procedures for both fatigue life prediction and fatigue life substantiation. Methods of calculation based on cumulative damage hypotheses are at present so uncertain that they cannot be recommended unconditionally [1] . Contrarily, experimental fatigue life substantiation is gaining more and more reliability mainly due to servo-hydraulic fatigue testing systems, which, partly computer controlled, keep pace with technological advance. However, it is assumed that in simulating the strain-time history and environmental conditions no deviations are allowed to occur, the effect of which on fatigue life would remain unaccounted for. In most cases, this condition cannot be fulfilled in the time available for a fatigue life substantiation. As a consequence, more or less arbitrary simplifications are made which raise doubts as to the reliability of the result. Even if simplification in individual cases can be justified by improved knowledge concerning the effect of particular parameters of simulation, a number of basic doubts will remain. This is so because at present there is no theoretically or experimentally established criterion available that would enable results obtained by testing individual components to be applied to other components employed in the same functional role, but of different design characteristics. Thus, a result, even if well defined with respect to mean value and scatter, is in most cases restricted to the component investigated. Therefore, a working hypothesis needs to be developed, which takes account of material response, design configuration, and production features, in addition to the expected strain-time function representative of a given service condition. The individual influence of each of these parameters must be accounted for, and also their mutual interaction. Only by team-work and by a common, well coordinated programme can this goal be reached efficiently within a foreseeable time.

The present contribution should be seen as a first step in this direction, a concept for a feasible procedure for solving current problems. In keeping with the predictive nature of the concept, it will be refrained from quoting results from literature or from current research work, because these are only in exceptional cases [31; 37] well enough defined so as to be used for supporting unambiguous arguments for or against the proposed concept.

2. BASIC CONSIDERATIONS

If one defines "fatigue life" as an integrated answer of a material, shaped and pretreated for its final, practical use and subjected to a determinate and/or random sequence of strain conditions of different complexity, including if necessary the environmental influences, then for reliable life prediction extensive basic information is necessary. However, in a satisfactory manner this is available for idealized conditions only, which in most cases do not apply. In future, essential parameters will have to be classified by fundamental considerations and assembled into a few parameter-combinations, which should occupy a central position within possible ranges of application. This procedure seems to be justified although it deviates from that used otherwise in research, where it consists in a systematic variation of individual, fully isolated variables. Moreover, based on experience, this procedure appears to be inescapable if one is not prepared to admit defeat in face of the great number of parameters and of what each individual research worker might consider as a desirable range of parameter bandwidth. Inherent in the problem is the natural scatter of life results. Therefore, parameters that are representative of service must enter into the laboratory test so that conclusions arrived at from relatively small samples retain their validity for larger series.

Figs. 1 and 2 show the increase in numbers of parameters due to transition from the constant amplitude tests to variable amplitude tests. Methods of simulation shown in Fig. 1 become constituent parts of certain parameter combinations, in which each parameter has its own bandwidth representative of service conditions.

These parameters are:

material characteristics,
specimen geometry and dimensions,
surface condition due to manufacturing process,
type of loading and environmental conditions.

The above reasoning stipulates parameter combinations which on the one hand lead to self-contained results, on the other are so related to each other that they can be considered as basic elements of a simple and reliable life prediction process.

They refer to

- the material, in the form of a typical design element, and to a well defined and easily obtainable strain-time function (see Section 3)

- generally used components and joining elements made of typical materials and subject to the same strain-time function as above (see Section 4)

- well defined strain-time functions considered as typical, and to a generally used design element made of typical engineering materials (see Section 5; Fig. 6).

If fatigue life prediction based on the aforementioned parameter combinations are to be confirmed by life substantiation tests, a number of further points need to be considered and subjected to closer scrutiny in each particular case (see Section 6). Several important points concerning the techniques of simulation and testing are discussed in Section 7.

3. MATERIAL

Material criteria for the suitability of design members subjected to random loading are, naturally, the usual strengths and technological data, further data concerning relevant fatigue behaviour, crack initiation, crack propagation, and residual strength data as a measure of fracture toughness. S-N curves are to be excluded, as in this special case the stress-time histories consist of constant stress amplitudes. This applies particularly to cases in which for a given design component, made in different alternative materials but otherwise under the same conditions, different slopes k are obtained (see Fig. 3). At present, it is not possible to substantiate any preference for a material having higher fatigue strength in the finite life range, as against one with a higher fatigue limit. Mostly, the usual cumulative damage hypotheses also remain unsuccessful as they fail to take account of the steady drop in fatigue strength, which sets in rather early for many materials [2]. Here, the only criterion of assessment is in the integral response of the material to a correctly simulated or a genuine random load sequence.

Engineering materials to be included in the above mentioned systematic comparison should be ones generally used in fatigue critical designs in various areas of application (mechanical-, civil-, automotive-engineering, etc.), having typical static strength properties appropriate to their use. Priority treatment should be given to materials in specific groups (e.g. heat-treatable steels) which are typical with respect to cyclic strain-hardening or strain-softening [3; 4].

3.1. Specimen configuration

The specimen configuration best suited for comparative investigations is based on the following reasoning. The fatigue strength of components, with all the behavioural aspects involved, is determined by interactions of elastic, plastic, and elasto-plastic deformations. Therefore, attention must be paid to the state of strain, strain concentrations and strain gradients, also as they change with time including strain redistributions due to local yielding. These interactions are tied up only with notched specimens representative of the design, so that unnotched specimens cannot be used for the comparison intended. It is recommended to choose two typical design elements of wide applicability having strain concentration factors of such magnitudes that the scatter bands obtaining do not overlap at all. Fatigue loading should produce in the element with the higher strain concentration factor a multi-axial state of strain (in-phase principal strains). For subsequent interpretation of test results and for obtaining basic data, it is of importance to measure and record the maximum strains occurring at the root of the notch in all specimens, and to note their variation with time until stabilization sets in. These data are to be evaluated in terms of mean value and scatter [5]. In simple cases, it appears that local strain is a better criterion of fatigue behaviour than is the nominal strain ε_n [6], especially so, if the strain gradient and the strained volume are used as concurrent parameters. This leads not only to better understanding of the mechanism of fatigue, but also results in considerable reduction of data.

At the Laboratorium für Betriebsfestigkeit, for 94 programme tests (see [8; 9; 10]), local strain measurements in the elastic range were carried out on motor car stub axles in nine different heat-treatable steels (nominal ultimate strengths between 700 and 1 300 N/mm^2, strain concentration factors between 1.2 and 1.8) of equal mean stress sensitivity [11], the evaluation of which [7] resulted in very narrow scatter bands for both the 50% and 90% probabilities of survival (Fig.4). Based on a known relationship between ultimate strength and fatigue life [12], the results were "normalized" to a nominal ultimate tensile strength of 850 N/mm^2 and transformed into a uniform (uniaxial) strain value $\bar{\varepsilon}_{a,f}$. Incidentally, this process lead to an unusually narrow scatter band of about $\pm 5\%$. The transformation is based on an empirical relationship between the elastic strain concentration factor $\alpha_{\varepsilon,el}$ and a fictitious fatigue notch factor $\bar{\beta}_{\varepsilon}$,

$$\bar{\varepsilon}_{a,f} = \bar{\beta}_{\varepsilon} \cdot \varepsilon_n$$

The factor $\bar{\beta}_{\varepsilon}$, depending on the strain gradient, takes account of cyclic deformation up to the maximum value in a given strain-time history. Life prediction could be considerably simplified if by testing design elements with various known strain concentration factors $\alpha_{\varepsilon,el}$ a relation could be established between the locally measured maximum value $\bar{\varepsilon}_a$, after stabilization has taken place, and the strain to failure $\bar{\varepsilon}_{a,fail.}$, determined under the same conditions (equal strain gradients and strained volumes). Herein, a "technical crack", i.e. a crack that can be recognized without optical aids, is assumed to be the failure criterion. This criterion is to be used especially in connection with components which are not of fail-safe design. Moreover, it is useful for a general characterization of materials, if amplified by data concerning crack propagation and area of final fracture. Unlike information derived from constant amplitude crack propagation tests, reliable data concerning "critical crack length" for random loading cases are not yet available.

3.2. Strain - time history

Parameter combinations concerning the essential variable "material" would not be complete without selecting a random load sequence to be used in fatigue tests for comparative life determination; see Fig. 1. It should be taken into account that random loading is not only due to vibration phenomena, but also to a sequence of single events, e.g. those which recur with each duty cycle in machines [13] . Both types of loading can be accounted for by the method of level-crossing counting [9] . Non-stationary frequency distributions occurring in service can often be expressed, on a theoretical basis, in terms of distribution functions prevailing under "stationary" conditions [4; 10] . For reasons connected with testing (to be dealt with in Section 7) design orientated material comparisons should be based on a stationary Gaussian process, the generation of which meets with no difficulties. The frequency bandwidth of the process should be narrow [3] , the power spectrum single-peaked, its intensity (RMS value) and the irregularity factor (H_0 / H_p) should be kept constant throughout the test. In generating this process, no mechanical or electrical limitations of extreme values should be allowed, not, at any rate, below about 5,5-times the RMS value (see Section 7). This recommendation intentionally breaks with the past, when it was left to the discretion of the individual research worker to simulate random loading sequences in a simplified manner, within limits dictated by the capability of the testing equipment available. The effects of these simplifications were not known then, and are even to-day not fully known. The above recommendations, or one similarly conceived, although less representative of service conditions than the often proposed quasi-stationary process with a multi-peaked power spectrum and a wide frequency bandwidth has the remarkable advantage that the testing time is reduced to about 1/10 of that required for quasi-stationary tests. The statement to the contrary in [14, Fig. 10] is tied up with the choice of the RMS value as a criterion for life resulting from different random processes, instead of taking the correct criterion which is the maximum strain.

4. DESIGN GEOMETRY, PRODUCTION AND ENVIRONMENTAL CONDITIONS

4.1. Aspects for material selection

Due to great differences in design geometry and in production processes, a life estimate in a particular case will remain unsatisfactory, until a strain criterion of some wider generality is found which might be more or less in line with the stipulations of Section 3.1. (compare with [15; 16] for fatigue life prediction of notched specimens under constant amplitude loading). In order to advance matters, design elements of general applicability should systematically be investigated under widely different and well defined strain conditions. These elements should be derived from the basic forms described in Section 3.1., and Fig. 6.

and be representative of production features associated with engineering components. If, for material groups of like response, this procedure results in a reliable strain criterion, then, in an actual design, not necessarily restricted in size, life can be determined by using a sample taken at random for assessing the area of validity of the relevant criteria.

For the design elements considered, a systematic dimensional variation, to account for the size effect, does not appear to be sensible as such elements are generally subject to high utilization when under random loading and therefore, due to the extreme values of the strain-time history, local yield points will be exceeded. Consequently, unlike for elastically strained members, there is no size effect to speak of. Production effects are disregarded as it is not readily possible to separate them from the size effect. However, the extent of strained volume and its effect on life and scatter will have to be considered. Up to the point of crack initiation it should, after all, be immaterial whether or not the randomly strained volume element is situated in a large or a small component, as long as all other conditions (i.e. state of strain, strain concentration and gradient, grain structure and technological effects) are equal. This reasoning may be used for determining the location of material extraction for the basic design elements to be made.

4.2. Load-transmitting design members

For combined, load-transmitting design elements, the state of strain of which cannot be calculated or ascertained by measurement (because it may depend on the type of load-transmission, or friction, fretting etc.), life prediction is only possible on the basis of prior experimental investigations, using specimens nearly identical with the final design. As a rule, riveted or bolted joints should be so optimized [17] by basic testing that they can be fitted into the final assembly without having to make changes. For experimental life substantiation of the complete assembly, this should lead to a high probability for avoiding premature failure of the joint. Welded joints are to be dealt with according to Section 4.1.

4.3. Production process

With due regard to considerations in Section 2, likely production methods (from semifinished product or blank, via heat treatment to final machining) should be typical for the design elements selected, so that all the possible factors are captured which have influence on the scatter of fatigue life. Dimensional variations are automatically eliminated by making local strain measurements on each individual member (especially important with regard to unmachined castings), see Section 3.1.

Concerning the effect of special production processes on the fatigue life of both simple and combined design elements (e.g. surface rolling, shot peening, interference-fit pins or bushes), the same applies, in principle, as stated in Section 4.2.

4.4. Environmental conditions

Reliable conclusions as to the effect of environmental conditions on fatigue life can only be arrived at by chronologically correct co-ordination with the strain-time history applicable to an individual case; they cannot be arrived at by using the "standard" random processes proposed here for basic investigations. Whether expectations are realistic with regard to temperature and corrosion resistance for a chosen material and surface treatment can only be proved or disproved by performing a so called "service duplication test" on an actual design member. For a qualitative approach to this problem, a parameter combination appears to be suitable in which the essential variable refers to the material. It is to be expected, that corrosion , much in the same way as fretting, is far less likely to cause strength reduction under random loading, than it does under constant amplitude loading in the vicinity of the fatigue limit [22] .

4.5. Parameter combination

The parameter combination discussed above should refer to engineering materials of wide applicability, the cyclic strain response of which is considered to be typical (see Section 3).

For the selection of a suitable random loading sequence, the criteria covered by Section 3.2. apply; especially so, if with increasing specimen size the deflections, and therefore the time required for testing, increases. This represents a link-up with the first parameter-combination on the basis of material as the essential variable.

5. STRAIN - TIME HISTORIES

5.1. Origin and simulation

Methods of experimental fatigue life determination are based on the fact that, in principle, two categories of strain-time histories are to be distinguished, due to different origins. As stated previously in Section 3.2., one strain-time history, on which attention has been focused recently, is based on the response of an oscillatory system to wide band random excitation, the other is based on that part of the duty cycle of a machine which, as a rule, can be described as a random sequence of single events [13] . Whilst in the former case, analysis can fully be substantiated by statistical data derived from system analysis, only few attempts in this direction have been made with respect to the latter. In the past three decades, marked by extensive efforts to solve the problem of life determination under simulated service loading, a method for analysis of strain-time history was used which is confined to counting the number of times defined threshold levels are exceeded [26] . This method has the great disadvantage that the resulting frequency distributions preclude any conclusions with respect to the chronological sequence of events. For a long time it was held that service conditions could only be simulated if the level-crossing spectrum was converted into a so-called amplitude spectrum, and this was vindicated by having to make use of testing facilities as then available [8; 20] . Hereby, it was also possible to evade the issue of defining what should be taken as "amplitude" in a strain-time history with a high irregularity factor when using one of the conventional cumulative damage hypotheses for life prediction.

With the introduction of servo-hydraulic actuator and of magnetic tape recording of service loading and its analysis by computer, simulation of any arbitrary strain-time history was made possible. However, the cumulative frequency curve of level-crossings, supplemented by certain statistical data, will certainly continue to play a significant role in defining strain-time histories as well as in experimental and computational assessment of fatigue life.

Special equipment provides,for servo-hydraulic testing machines,the input of all characteristics of an oscillating system, so that a design element (taking into account the dynamic response of the control system of the testing machine) can be investigated under practically the same strain-time history as imposed in service. If even minute deviations are to be eliminated, command value signals recorded on magnetic tape may be used directly for controlling the testing machine (service duplication test). The latter procedure has the advantage that for fractographic investigations a correlation of crack length with the test load is feasible at every instant in time during a test.

It appears that the use of modern equipment for simulation of random processes [19] has resulted in a trend toward emphasising randomness, and to neglect other parameters having decisive influence on fatigue life, which are then not evaluated numerically and not monitored for constancy. Thus, generally it is rather difficult to compare results obtained from different research institutes using different parameters and procedures for simulating random processes. The diagram referred to earlier (see Fig.2) shows the high number of parameters for simulation or generation of a random process that can be freely selected. As yet, reliable information is not available as to the significance of individual variables, allowing for reduction of results from one parameter-combination to those of another parameter-combination. Based on these considerations, a procedure appears to be suited for experimental fatigue life determination which stipulates a few, easily and reliably reproducible "basic types" of random processes, modelled on those occuring in service.

5.2. Basic strain - time histories

The Gaussian random process is proposed as a basic type of a random process which, because of its freedom from ambiguity, proves to be an analogue to the deterministic strain-time function having constant amplitude (CA, Fig.1). The proposed basic type is to be a stationary Gaussian process with single-peaked power spectrum (SGR/1) and relatively narrow bandwidth (see Fig.1), the same as intimated in Sections 3 and 4 for the parameter combinations "material" and "design". This basic type of process has the great advantage that the fatigue results it yields practically do not differ from those obtained by a Gaussian normal distribution, being a cumulative frequency distribution for random single events [10] . Modifications having influence on both these types of random processes, be it due to functional or other limitations of extreme values, are in their effect easily assessable [20; 21; 27] .

In a logical sequence of basic types of random processes for a practical range of application, the single-peaked, narrow-band stationary Gaussian process

(SGR/1) is followed by the stationary Gaussian process with double-peaked power spectrum (SGR/2, Fig. 1) and extended bandwidth. The quasi-stationary random process with double-peaked power spectrum (QSR, Fig. 1) and the same bandwidth as type SGR/2 affords the widest range of application and completes the sequence of strain-time functions within the third parameter combination for fatigue life determination. This type can be considered as equivalent to a sum of stationary Gaussian processes SGR/2 using the same distribution of power spectral density, but RMS values of different magnitude and with different duration [4; 23; 27] .

As compared to the other two basic types, the quasi-stationary random process has the disadvantage that, for the same maximum value of the cumulative frequency distribution (e.g. $\bar{\varepsilon}_a$), testing time will be about ten times longer. Thus, computation appears most necessary for predicting fatigue life, especially when it concerns this and similar random processes.

With respect to strict comparability of results from different fatigue tests, the following parameters for the proposed three basic types are of particular significance:

- Concerning the stationary Gaussian process SGR/1

  distribution of the single-peaked spectral power density; this contains the RMS value of the random variable, (e.g. ε_{rms}), centre frequency f_c ; bandwidth Δf and irregularity factor $I = H_0 / H_p$.
  Further, the so-called crest factor C, (i.e. the ratio of the maximum value, e.g. $C = \bar{\varepsilon}_a / \varepsilon_{rms}$), has to be determined by measurements made during the test.

- Concerning the stationary Gaussian process SGR/2

  distribution of double-peaked power spectral density, RMS value, the two centre frequencies, bandwidth, irregularity factor and crest factor.

- Concerning the quasi-stationary random process QSR

  distribution of double-peaked power spectral density, having the same shape as SGR/2, but with randomly varying RMS value. The resulting distribution of the power spectral density and associated mean RMS value, (referring to the whole test duration) the mean irregularity factor and the crest factor are to be determined and stated.

The three basic types of processes were based on reasoning - similar to that in [20] - that, in going from one process to the next, only one parameter essentially of influence on fatigue life should be alterated, namely the shape of the power density distribution. Hereby, at the same time, prerequisites for verification of an "extended" cumulative damage hypothesis [24; 25] are given. This enables reliable assessment of fatigue life to be made using Type QSR, resolved into its stationary components of Type SGR. Partial damage pertaining to each SGR component can be calculated using the appropriate fatigue life curves (see Fig. 5). Recommended numerical values for individual parameters of the three basic types are given in Section 7.

5.3. Parameter combination

The parameter-combination based on "strain-time function", as discussed here, should be applied mainly - as in the case of the parameter-combination "design .."- where it concerns widely used materials exhibiting typical cyclic strain response and specimens conforming to 3.1.

6. FATIGUE LIFE SUBSTANTIATION

For substantiating the conclusiveness of fatigue life determination based on results of the three parameter combinations, specific considerations apply. Generally, as the interrelationship material, geometry, production, and strain-time function of the test specimen is not identical with the parameter combination of one of the basic processes, substantiation tests are required, at least on the basis of taking samples. These tests should be based on the original design, e.g. a prototype, and on the strain-time functions representative of those that can be anticipated or have been measured in service. These are then to be duplicated without simplifications.

With constructions having a vital role to play and only one fatigue critical location in which strain is definitely associated with one external load always acting in one direction, there should be, under no circumstances, any departure from the "service loading duplication test" [3] , which has the added advantage that it enables environmental conditions to be simulated most realistically. What is said in Section 3.1. concerning local strain measurement must be taken into account, as only then can results from substantiation tests be considered as contributory criteria for the applicability of the concepts proposed. If a fatigue life assessment and the result of a substantiation test diverge considerably (ratio greater than 1:3), and if it is supposed that deviations are attributable to peculiarities of the strain-time function, it is advisable to conduct a series of tests under a random process of the type SGR/1. If, on the other hand, the design shape exhibits peculiarities, a service loading duplication test should be meaningful, using a strain-time pattern of the parameter combination "design ..." which is in its effect comparable to that of the design at hand.

Where it concerns large assemblies or full-scale structures having numerous fatigue critical locations or areas which are not predictable in advance, life substantiation by means of a representative service loading duplication test becomes much more difficult - if not altogether impossible. Generally, the aim is to reduce the time-varying strains present in these locations to several simultaneous or intermittent external loads of different direction and origin, vibrations or single events. As yet, for lack of relevant information [28] , it is not possible to vary and correlate these loads chronologically so as to result in a test that would be truly representative of service loading duplication. More or less simplified, "constructed" tests of this kind might, in some locations, achieve a response equivalent to that of service loading. However, in other locations there might be no response, or it might be misleading, so that the value of such tests remains doubtful. In planning service loading duplication tests, it should be remembered that it is seldom possible to simulate the dynamic response of larger structural areas through quasi-static single load inputs (which is often the only feasible loading). Whether and to what extent the full-scale fatigue test can be replaced by tests on a number of small fatigue-prone areas might be the subject of a future paper, which would also have to deal with the possibility of testing a number of identical sections for determining scatter of lives. This type of testing must rely for better simulation of local strains on static and dynamic strain-time measurements performed on the complete structure.

7. SIMULATION- AND TESTING-TECHNIQUES

7.1. Typical random processes

To assure comparability of test results pertaining to random loading and using the proposed types of vibration processes, care should be taken that the design members tested answer with processes,the statistical data of which correspond to the following guidelines.

- Single-peaked stationary Gaussian process SGR/1

  A desire for short test duration and the seemingly small influence of the centre frequency f_c [29] would imply the use of rather high test frequencies. This is contradicted by the fact that the cyclic deformation of materials in the plastic range is frequency-dependent. Therefore, a centre frequency f_c considerably higher than that occuring in service should be avoided. As long as no reliable results are available, it is recommended to use a centre frequency between 10 and 30 cps for the basic type SGR/1.

Judging by results obtained from several series of conclusive tests on a single material (Al Cu Mg) [30] , the irregularity factor $I = H_0 / H_P$, determined by the shape of the power spectral density and the bandwidth Δf, seems to have little influence on life [20; 31] . Therefore, selecting an irregularity factor of $I \approx 0.7$ appears to be reasonable as a mean between $I \approx 0.3$ [32] , a value observed in practice, and $I \approx 1$, which occurs seldom in practice. According to [35] , this choice appears to be acceptable also from the test equipment viewpoint (combination of servo-hydraulic testing machine, random generator, and filter characteristics).

The magnitude of the RMS value of the random variable determines the life of a given design member subject to random loading, which in turn is defined by the power spectral density distribution [29; 30; 34; and others] . To establish a fatigue life curve (see Fig. 5) according to proposals in [10] , several test series should be used, each consisting of six to eight specimens at least. The RMS value in each of the series is to be constant, but suitably varying from series to series. In the test series having the highest RMS value, specimens should sustain on average $2 \cdot 10^6$ mean crossings ($\bar{N}_0$) to failure. For a sound design, this represents a figure of little interest, as the usually expected value of $\bar{N}_0$ under service conditions lies between 10^7 and 10^9.

In planning simulated service life tests under random loading, the following points require attention:

If in test series used for determining the fatigue life curve, the mean stress differs from zero, the ratio $\bar{R}$ of upper to lower extreme value for the individual series must remain constant (extreme value = C · RMS value) [33] according to the requirements of design practice. Otherwise it is impossible to define systematically the fatigue life curve by the exponent $\bar{k}$ [25]. Concerning the significance of the RMS value as a criterion of loading, some reservations must be made. On the one hand, the RMS value cannot be applied in the same sense (as an expression for the mean square value of the time domain) to loading of different origins(see Section 5.1.)[1]; on the other hand it is not sufficiently sensitive in respect of infrequently occurring high random loads, although these have a decisive effect on life. As fatigue life curves are used by the designer to determine the allowable strain for a required life, RMS values should be replaced or complemented by a better and clearer design criterion, e.g. by an extreme value of the cumulative frequency distribution, which on average occurs once per 10^6 mean crossings [10]. By stating a defined extreme value, it is easier to establish the available margin with respect to strain limits which must not be exceeded. If in ignorance of the extreme value to be expected this in fact turns out to be too high, the 0.2-proof of the material might be exceeded, thus restricting or invalidating the results obtained by random loading tests [31]. Therefore, it is recommended to record the response of the design member to the simulated random process, or at least the level-crossing frequencies experienced by it, and state the cumulative frequency distribution and the crest factor C with the test result, e.g. as in [31]. In programme testing $C \approx 5.25$ has been found to be an acceptable value [10; 30].

---

1) With simplifications, introduced at simulation, e.g. of the wave form of strain-time history, the RMS-value changes, but not the material response.

- Double-peaked Gaussian process SGR/2

Judging by the few PSD distributions which were published, it appears reasonable to use centre frequency ratios $f_{c1} / f_{c2} \approx 0.01$ to 0.05, wherein f_{c1} should be between 0.5 and 2.5 cps. According to Bussa [31], this ratio has no significant influence on life.

Concerning the shape of the power spectral density distribution and the bandwidth, it appears to be expedient to choose them so that an irregularity factor $I \approx 0.7$results. With respect to the RMS value, the same applies as in the case of the single-peaked SGR/1.

- Quasi-stationary double-peaked random process QSR

The shape of distribution, centre frequency and bandwidth are the same as with the two-peaked Gaussian process. Therefore, the irregularity factor is also the same. Relative to the preceding basic type, the difference lies in the RMS value which does not remain constant, but changes according to the probability density function. This was calculated by Jaeckel and Swanson [14] and found by Buxbaum [32] , evaluating representative long-time records.For simulating in a test the density function of RMS values, this function is suitably stepped and used together with the associated time intervals (proportional to probability density) as a random sequence of RMS steps. Whether the sequence of RMS values is random or determinate is probably of no account, as the random character of loading is not changed, only its intensity is. For this random process, as for the others, it is strongly recommended to record the specimen response continuously, to evaluate the average RMS value, the crest factor, the irregularity factor, and to enter them into the test log.

7.2. Testing technique

The testing technique for simulating random processes has as yet not reached a level justifying generally valid recommendations beyond those contained in Section 7.1. The use of random generators, in conjunction with appropriate filters, for generating the command value input to control servo-hydraulic testing machines depends on the following conditions:

- the random generator must be capable of generating extreme values having a magnitude at least 5.5 times that of the RMS value,
- it must provide a frequency distribution which corresponds exactly to the Gaussian process,
- this frequency distribution must be symmetrical about the linear mean value of the random variable.

Even if these requirements are fulfilled, certain drawbacks are inherent in this method of generation; experience has shown that, depending on the number of mean crossings to failure, cumulative frequencies differ widely, especially in the range of the extreme values. This circumstance tends to increase scatter of test results which makes results non-representative, whereas in service scatter diminishes with increasing service life. Moreover, large scatter is undesirable with respect to the information value obtained from comparative test series.

These drawbacks can be obviated if the process required is generated by means of a computer and used, so to speak, as a deterministic sequence of loading representing the command values. To equal effect, the response of an oscillating system having a given frequency response can be recorded on magnetic tape and used as the command value input [3].

With regard to investigating the initiation and propagation of a crack, this procedure, as already mentioned in Section 5.1, has the advantage that throughout the test, the magnitude of the applied load has a known value at every instant in time. If input is obtained from a computer or magnetic tape, it is possible to insert at will relatively high loads occurring in a spectrum; their effect is then revealed in the fracture micrograph (marker loads), whereby the plotting of crack propagation curves is greatly facilitated.

With both methods of generation, a process can be extended over such a period that extreme values containing the proposed crest factor $C \geqslant 5.5$ are with certainty included in the spectrum. For this value of C a very good approximation to a Gaussian frequency distribution will be obtained. Of the total time over which the process was generated, a representative section with about $2.5 \cdot 10^5$ to $5 \cdot 10^5$ mean crossings is extracted and, as a sort of "random block" [10], repeated until the specimen cracks or fails [3]. This guarantees the lowest scatter of test results [38].

8. CONCLUSIONS

The "block programme test" (constant amplitudes at eight levels, [9; 18; 36]) developed by the author between 1938 to 43 for the purposes of aircraft design [8] and introduced into automotive design in 1948, has proved its worth in applications such as the determination of allowable stresses and life substantiation for design members subject to varying loading. The drawback inherent in this test method lies in the difference between the programmed sequence of constant amplitudes as applied in the test and the random loading sequence experienced in service. Nevertheless, for over two decades, the design and life substantiation of new aircraft and automobile types and of new machines, has been based on results obtained by block programme testing, with no setbacks having been reported. Thus, the proposed concept, using random loading tests and an improved strain criterion backed by experience resulting from hundreds of conventional block programme tests should lead to a more reliable method of life prediction. Consequently, life substantiation required for clearing a design for series production, or for official certification, would serve mainly for verifying that the design is well dimensioned and that major design changes are not called for.

The recommended parameter-combinations, primarily intended for application to practical cases, might serve also as a "framework" for tackling research problems of a more fundamental character. Thereby, questions necessarily left open at present could receive due attention.

This paper does not contain the item "Analysis of the Influence of the Irregularity Factor", the author reported about additionally. It will be sent as a separate publication to all participants of the 13th ICAF Conference.

9. REFERENCES

[1] Hardrath, H.F.:
Review of Cumulative Damage,
Vortrag AGARD-Meeting München (1965). NASA RM X-56749.

[2] Haibach, E.:
Modifizierte lineare Schadensakkumulations-Hypothese zur Berücksichtigung des Dauerfestigkeitsabfalls mit fortschreitender Schädigung.
Laboratorium für Betriebsfestigkeit, Darmstadt, TM Nr. 50/70.

[3] Gassner, E.:
Betriebsfestigkeit gekerbter Stahl- und Aluminiumstäbe unter betriebsähnlichen und betriebsgleichen Belastungsfolgen.
Materialprüfung 11 (1969) Nr. 11 S. 373/78.

[4] - :
Ziele einer praxisorientierten Schwingfestigkeitsforschung.
Schriftenreihe der Fraunhofer-Gesellschaft, München,
Heft 5 (1968).

[5] Schütz, D.:
Durch veränderliche Betriebslasten in Kerben erzeugte Eigenspannungen und ihre Bedeutung für die Anwendbarkeit der linearen Schadensakkumulations-Hypothese.
Laboratorium für Betriebsfestigkeit, Darmstadt, Bericht Nr. FB-100 (1972).

[6] Gassner, E., E. Haibach :
Die Schwingfestigkeit von Schweißverbindungen aus der Sicht einer örtlichen Beanspruchungsmessung.
In: Tragfähigkeitsermittlung bei Schweißverbindungen. Fachbuchreihe Schweißtechnik Bd. 53/1 (1967) S. 47/73.

[7] Seitz, E.:
Über den Einfluß der Gestaltung auf die Lebensdauer von Achsschenkeln.
Unveröffentlichter LBF-Bericht, Diplomarbeit TH München (1969).

[8] Gassner, E.:
Festigkeitsversuche mit wiederholter Beanspruchung im Flugzeugbau.
Luftwissen 6 (1939) Nr. 2 S. 61/64 (engl. Übersetzung:
"Strength Investigation in Aircraft Construction under Repeated Application of the Load", NACA TM 1087, (1946).

[9] - :
Betriebsfestigkeit, eine Bemessungsgrundlage für Konstruktionsteile mit statistisch wechselnden Betriebsbeanspruchungen.
Konstruktion 6 (1954) Nr. 3 S. 97/104.

[10] Haibach, E., W. Lipp:
Verwendung eines Einheitskollektivs bei Betriebsfestigkeits-Versuchen.
Laboratorium für Betriebsfestigkeit, Darmstadt, TM Nr. 15/65.

[11] Schütz, W.:
Über eine Beziehung zwischen der Lebensdauer bei konstanter und bei veränderlicher Beanspruchungsamplitude und ihre Anwendbarkeit auf die Bemessung von Flugzeugbauteilen.
Z.Flugwiss. 15 (1967) Nr. 11 S. 407/19.

[12] Gassner, E.:
Einfluß der Werkstoff-Bruchfestigkeit auf die Lebensdauer von Achsschenkeln.
ATZ 59 (1957) Nr. 9 S. 283/84.

[13] Buxbaum, O., O. Svenson :
Zur Beschreibung von Betriebsbeanspruchungen mit Hilfe statistischer Kenngrößen.
ATZ 75 (1973) erscheint im Juni.

[14] Jaeckel, H.R., S.R. Swanson:
Random Load Spectrum Test to Determine Durability of Structural Components of Automotive Vehicles.
In: Proceed. 12th International Automobile Technical Congress, FISITA, Barcelona (1968).

[15] Topper, T.H., R.M. Wetzel, J.D. Morrow:
Neuber's Rule Applied to Fatigue of Notched Specimens.
J.Materials 4 (1969) Nr. 1 S. 200/09.

[16] Wellinger, K., H. Gaßmann, J. Luithle:
Plastische Wechselverformung und Dehnungsformzahl.
In: VDI-Berichte Nr. 129 (1968) (engl. Übersetzung: Cyclic Plastic Deformation and the Strain Concentration Factor. LBF-Bericht Nr. TÜ-84, 1970).

[17] Schütz, D., O. Gökgöl:
Optimierung der verschraubten Fügung zwischen Flügelunterdecke und Anschlußbeschlag eines Militärflugzeuges.
Laboratorium für Betriebsfestigkeit, Darmstadt, Bericht Nr. TB-95 (1971).

[18] Gassner, E.:
Effect of Variable Load and Cumulative Damage of Fatigue in Vehicle and Airplane Structures.
In: Proceed. International Conference on Fatigue of Metals, London-New York 1956. Institution of Mechanical Engineers, London (1956) S. 304/09.

[19] Jacoby, G.:
Das Problem der Schwingfestigkeit im Kraftfahrzeugbau.
Sonderdruck C. Schenck Maschinenfabrik, Darmstadt (1972).

[20] Gassner, E., E. Haibach:
Testing Procedures for the Design and Life Estimation of Fatigue-Sensitive Structures.
In: Structural Safety and Reliability. Ed. by A.M.Freudenthal, New-York: Pergamon Press Oxford (1972) S. 299/321.

[21] Ostermann, H.:
Einfluß der Kollektivform auf die Schwingfestigkeit der Aluminium-Legierung 3.1354.5 (Al Cu Mg 2).
Laboratorium für Betriebsfestigkeit, Darmstadt, TM Nr. 36/68.

[22] Gassner, E.:
On the Influence of Fretting Corrosion on the Fatigue Life of Notched Specimens of an Al Cu Mg 2 Alloy.
In: Proceed. Ninth ICAF-AGARD-Symposium, Paris 1961. Ed.by W. Barrois, E.L. Ripley, Oxford, New York, Paris: Pergamon Press London, (1963) S. 87/95 (deutsche Übersetzung: Über den Einfluß der Reiboxydation auf die Lebensdauer gekerbter Proben aus einer Al Cu Mg 2-Legierung. Aluminium 39 (1963) Nr. 9 S. 582/584).

[23] Kowalewski, J.:
Beschreibung regelloser Vorgänge.
Fortschritt.-Ber. VDI-Z., Reihe 5 (1969) Nr. 7.

[24] Gassner, E., W. Schütz:
Assessment of the Allowable Design Stresses and the Corresponding Fatigue Life.
In: Fatigue Design Procedures, Proceed. 4th ICAF Symposium, München 1965. Ed. by E. Gassner, W. Schütz, New York: Pergamon Press Oxford (1969) S. 291/307.

[25] -:
Performance Fatigue Testing with Respect to Aircraft Design.
In: Fatigue in Aircraft Structures. Ed. by A.M. Freudenthal, New York: Academic Press Inc. (1956) S. 178/206.

[26] -:
Ergebnisse aus Betriebsfestigkeits-Versuchen mit Stahl- und Leichtmetallbauteilen.
In: Bericht der Lilienthal-Gesellschaft für Luftfahrtforschung 152 (1942) S. 13/23.

[27] Bussa, S.L., N.J. Sheth, S.R. Swanson:
Development of a Random Load Life Prediction Model.
Ford Component Test and Development Activity Report 70-2204 (1970).

[28] Jaeckel, H.R.:
Simulation, Duplication, and Synthesis of Fatigue Load Histories.
Society of Automotive Engineers, New York, Paper 700032 (1970).

[29] Sherratt, F., B.C. Fisher:
Extracting Fatigue Testing and Design Data from Service Loading Records.
In: Conference on the Recording and Interpretation of Engineering Measurements. Joint B ritish Committee for Stress Analysis. Ed. by The Institution of Marine Engineers, London (1972) Nr.2 S. 27/33.

[30] Kowalewski, J.:
Über die Beziehungen zwischen der Lebensdauer von Bauteilen bei unregelmäßig schwankenden und bei geordneten Belastungsfolgen.
Deutsche Versuchsanstalt für Luftfahrt, Mülheim, Bericht Nr.249 (1963).

[31] Bussa, S.L.:
Fatigue Life of a Low Carbon Steel Notched Specimen under Stochastic Conditions.
Ford Motor Company, M.Sc.Thesis, Wayne State University, Detroit (1967).

[32] Buxbaum, O.:
Beschreibung einer im Fahrbetrieb gemessenen Beanspruchungs-Zeit-Funktion mit Hilfe der spektralen Leistungsdichte.
Laboratorium für Betriebsfestigkeit, Darmstadt, Bericht Nr. TB-102 (1972).

[33] Talreja, R.:
On Fatigue Life under Stationary Gaussian Random Loads.
Department of Solid Mechanics, The Technical University of Dänemark (1972).

[34] Slevenson, S.A., R. Steiner:
Fatigue Life under Random Loading for Several Power Spectral Shapes.
NASA R-266 TR (1967).

[35] Haibach, E., H.G. Köbler:
Berechnung des Unregelmäßigkeitsfaktors N_0 / N_1 für einen stationären Gaußprozeß mit eingipfligem Spektrum der Leistungsdichte.
Laboratorium für Betriebsfestigkeit, Darmstadt, Bericht Nr. TB-97 (1972).

[36] Conover, J.C., H.R. Jaeckel, W.J. Kippola:
Simulation of Field Loading in Fatigue Testing.
Society of Automotive Engineers, New York, Paper 660102 (1966).

[37] White, D.J., J. Lewszuk:
Cumulative Damage in Push-Pull Fatigue of Fillet-Welded Mild Steel Plate Subject to Narrow Band Random Loading.
In: Proceed. 1970-71 Institution of Mechanical Engineers, London, S. 339/51.

[38] Lipp, W.:
Simulation von Betriebsbeanspruchungen im Nachfahrversuch.
Laboratorium für Betriebsfestigkeit, Darmstadt, TM Nr. 59/71.

10. NOTATION USED IN FIGURES

STH	Strain-time Function
PSD	Distribution of power spectral density
PSDS	Shape of power spectral density distribution
f_c	Geometric centre frequency
Δf	Ratio of limiting frequency (width of spectrum)
$\phi(f)$	Power spectral density
CFD	Cumulative frequency distribution, spectrum
cCFD	Continuous spectrum
sCFD	Stepped spectrum
H	Level crossing frequency
H_0	Total number of mean crossings with positive slope
CA	Constant amplitude
PCA	Programmed constant amplitudes
SGR/1	Stationary Gaussian process with single-peaked power spectral density
SGR/2	Stationary Gaussian process with double-peaked power spectral density
QSR	Quasi-stationary random process with double-peaked power-spectral density and varying RMS value
SD	Duplication of service loading
ε_m	Mean strain
ε_a	Strain amplitude
$\bar{\varepsilon}_a$	Maximum strain amplitude of a spectrum
$\bar{\varepsilon}_{a,f}$	Fictitious maximum strain amplitude
ε_{RMS}	Root mean square value of strain
RMSD	Power spectral density distribution of the RMS value
S_E	Fatigue limit for stress or strain,respectively
N_E	Number of cycles at the "knee" of the S-N curve
$\bar{N}_0$	Number of mean crossings to failure referring to strain
k	Exponent in equation for S-N curve

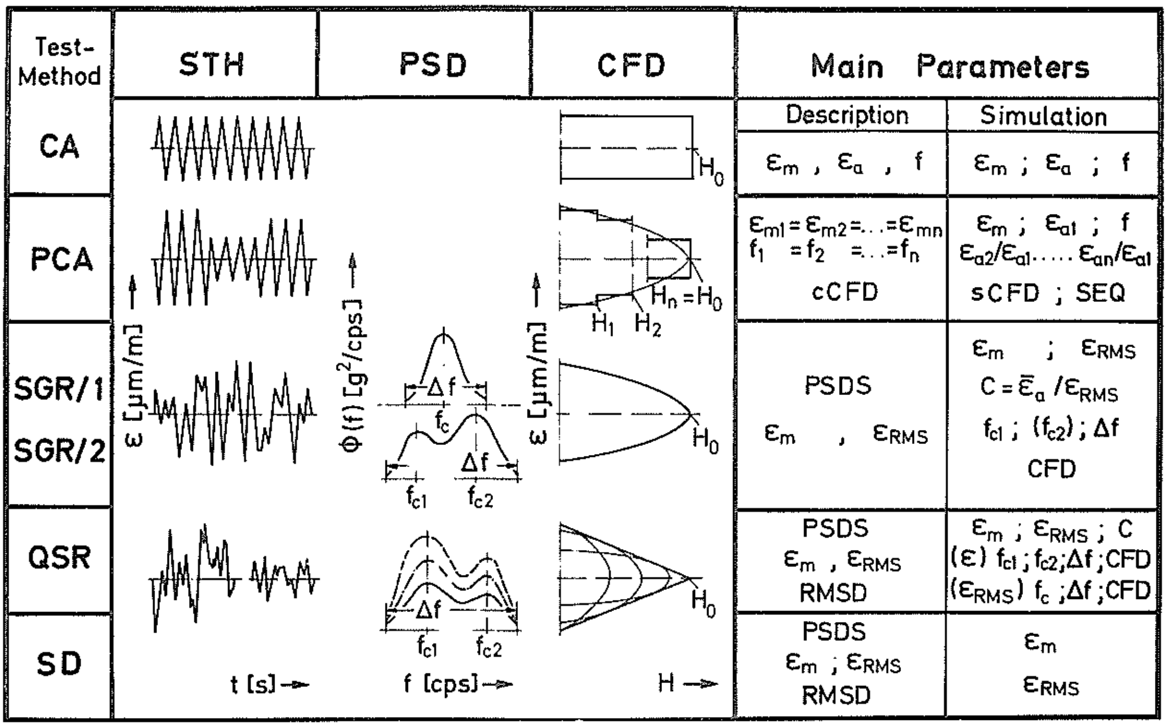

Fig. 1: Strain-Time Functions. Parameters for Description and Simulation

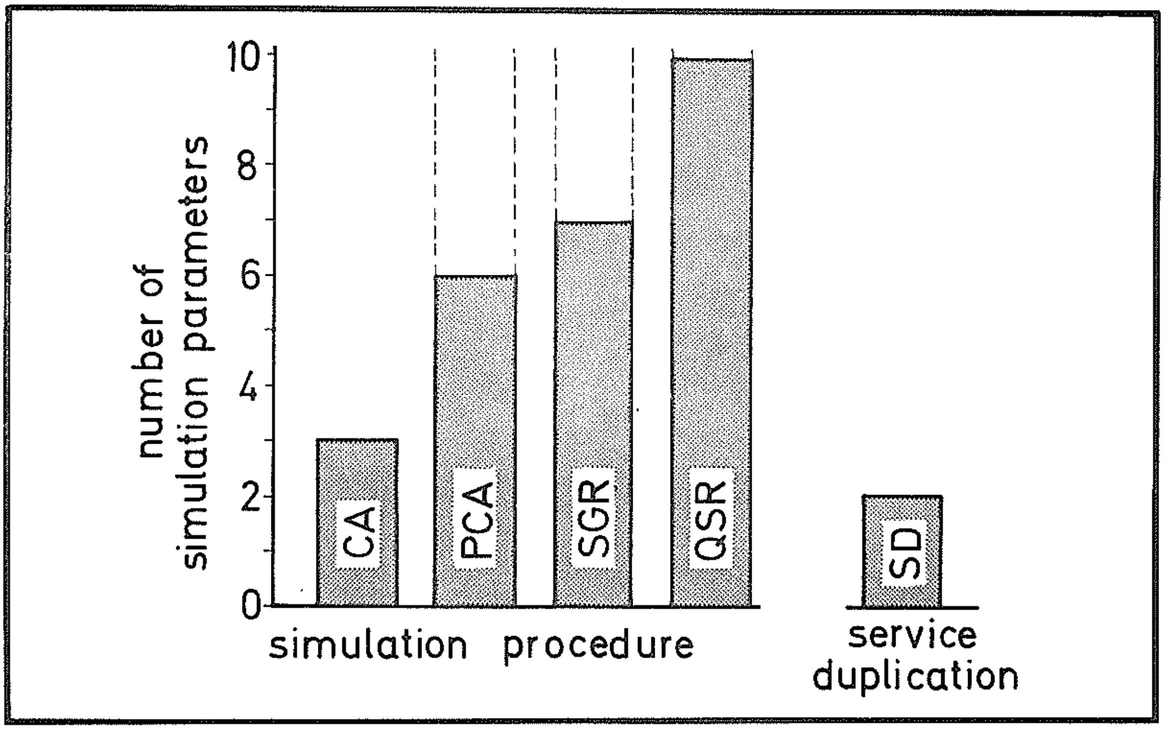

Fig. 2: Number of Simulation Parameters Required Depending on Degree of Realism with which Service Loading is Simulated

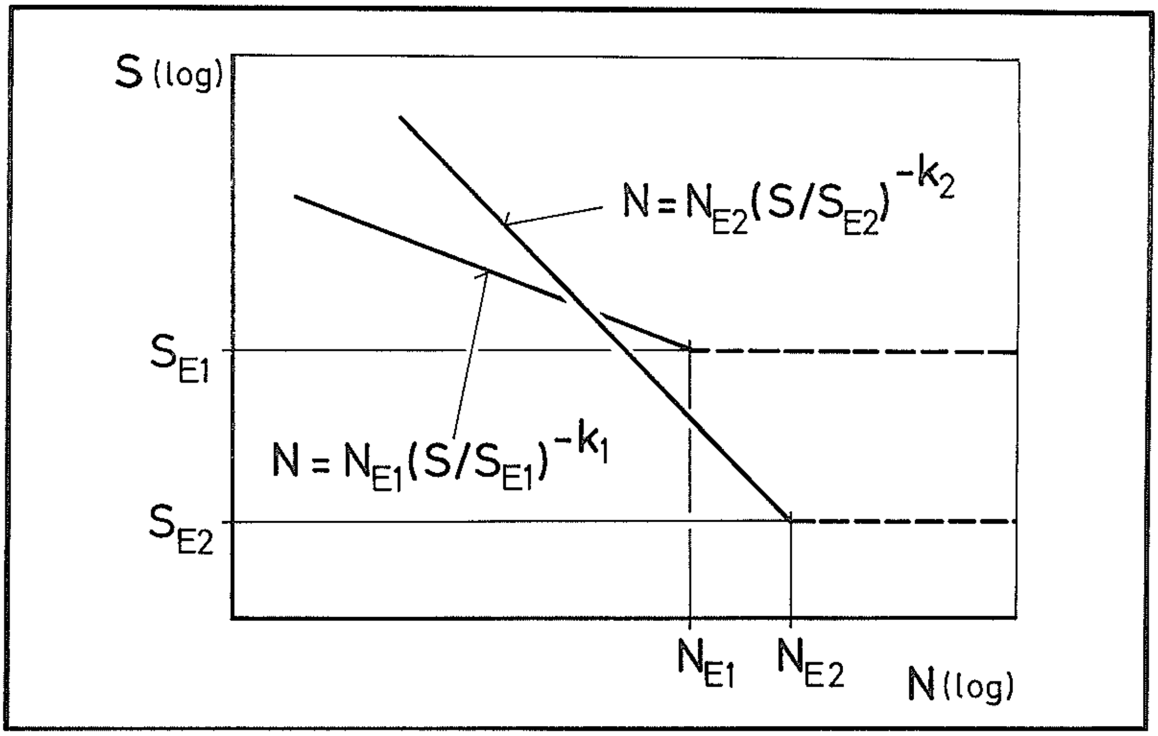

Fig. 3: S-N Curves of Different Slope k

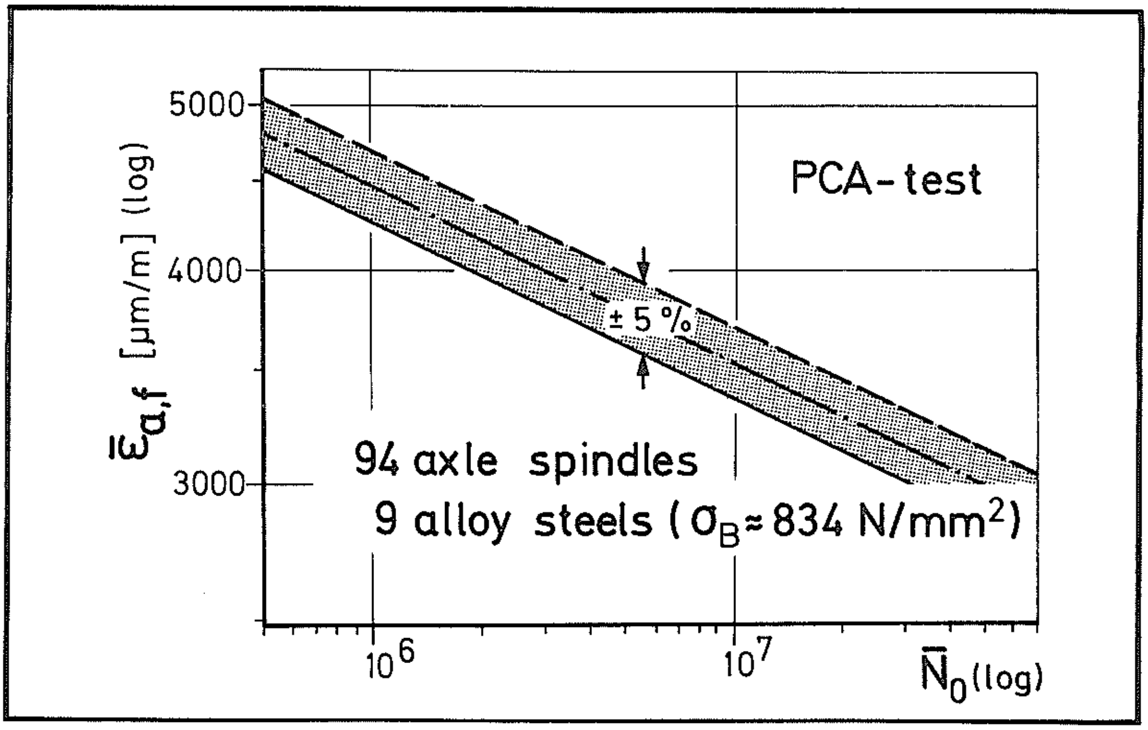

Fig. 4: Fictitious Maximum Strain $\bar{\varepsilon}_{a,f}$ as Function of Life

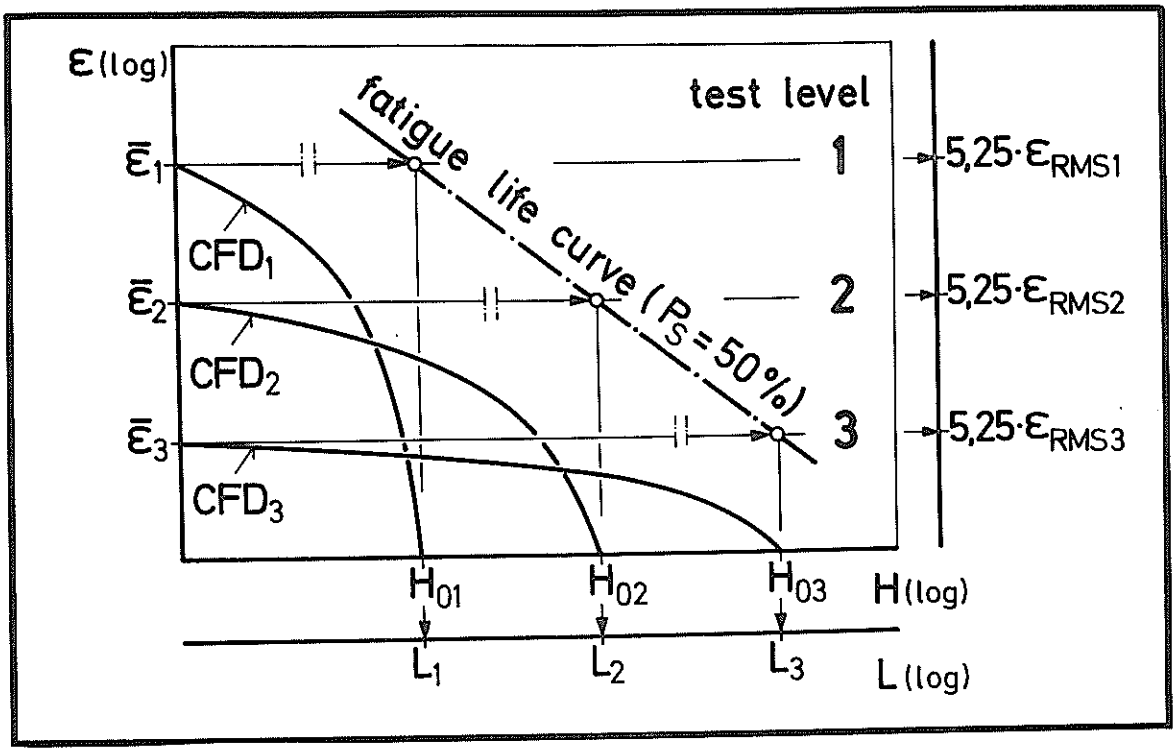

Fig. 5: Results of Random Loading Tests Plotted as Fatigue Life Curves

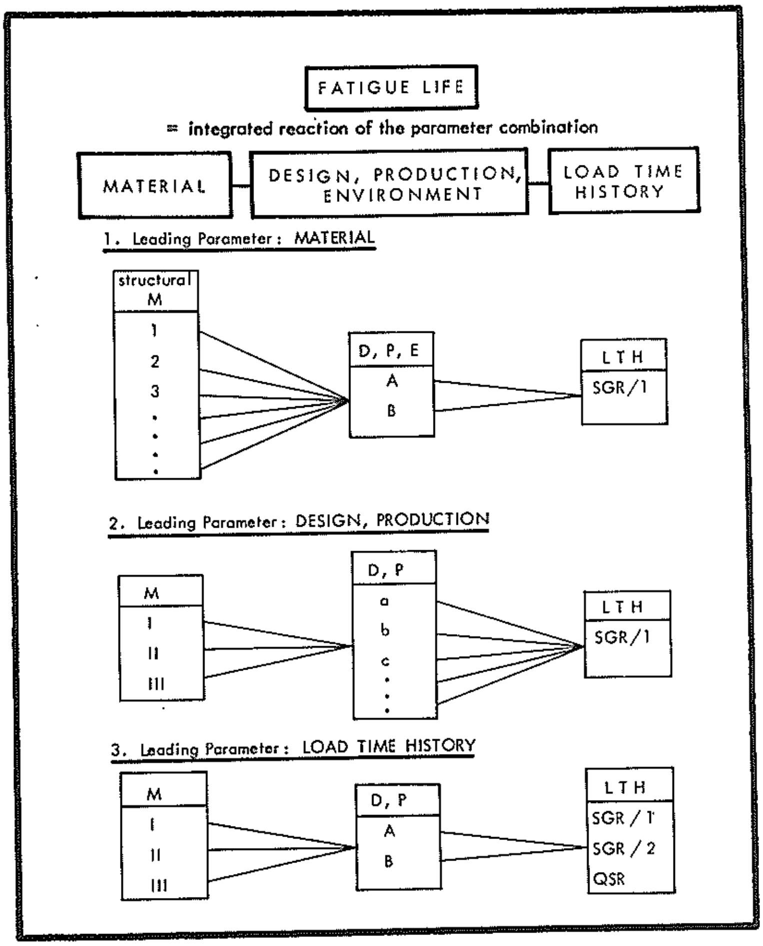

Fig. 6: Scheme for Planning Fundamental Tests for Life Determination