RELIABILITY ANALYSIS OF WING PANEL CONSIDERING TEST RESULTS FROM INITIATION OF FIRST AND SUBSEQUENT FATIGUE CRACKS

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ABSTRACT

The safety of an aircraft structure with regard to fatigue is usually considered adequate if the structure is capable of withstanding a fail-safe load of a defined magnitude at the end of an inspection interval, where a detectable crack has been present since the beginning of the same interval. This, or similar criteria, does not provide a uniform level of safety, however, as expressed by a given figure of the probability of failure that is often prescribed in modern design codes. In order to achieve such a safety goal it is necessary to analyse statistically the entire course of a fatigue failure, starting with a crack which is initiated and propagates due to the random loadings experienced in every flight during service life. The crack growth results in a continuous deterioration of the ultimate strength of the structure, and extreme turbulence may finally cause the residual strength to be exceeded by a heavy gust load. An existing crack of a certain length will normally be detected at the inspection, the probability of success depending on which method of inspection is chosen. In case the crack is not detected it will continue to grow adding to the risk of failure during the next inspection interval.

The present status of the reliability analysis of a fatigue-sensitive wing panel is described, discussing the possibilities to base the theory on realistic assumptions and to introduce in the numerical calculations satisfactory statistical data. Rather extensive information seems to be available concerning load spectra to be applied. Also the crack propagation and the residual strength has been subjected to successful theoretical and experimental investigations in the rapidly developing field of fracture mechanics. The crucial point is no doubt the lack of data of crack initiation times for realistic full-scale panels including the uncertainty involved in crack detection. Fatigue tests

to cure this lack are expensive and large sample sizes of identical specimens are not likely ever to become available. Interesting observations regarding scatter and distribution functions have been made recently by pooling large numbers of results from different investigations.

Tests carried out at FFA have included two series with about 20 panels in each. The panels had 32 similar stress concentrations and were fatigue cycled using a flight-by-flight load program. In the first test series the number of flights to detection of the first crack was determined. The crack propagation was followed to various crack lenghs and the residual strength was then determined. In the second series the detection of the first, second and third cracks was recorded and it was possible to pool all crack initiation times by computing the common "parent" distribution, i.e. the probability of crack initiation at a single stress concentration.

Using the information obtained from the FFA tests numerical calculations have been made of the failure risks for various lengths of inspection intervals and service lives. Although the test specimens were considered to simulate with some realism wing panels of a transport aircraft, the diagrams given should not be regarded as direct design information but rather as examples of a mixed analytical-experimental procedure to follow.

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FOREWORD

In May 1951 a letter was written from the Nationaal Luchtvaartlaboratorium, Amsterdam, to the College of Aeronautics in Cranfield, UK, and the FFA in Sweden, proposing a collaboration in the field of aeronautical fatigue between the three institutes mentioned. The father of this idea was Dr Frederik J Plantema, head of the Structures and Materials Department at NLL. Representatives of NLL, CoA and FFA met in Cranfield in September 1951, where an agreement was reached on the future activities according to the Dutch suggestions. It was also decided to invite participation from Belgium and Switzerland. The first Conference of the International Committee on Aeronautical Fatigue, ICAF, was held in Amsterdam in 1952.

The foundation of ICAF in the beginning of the fifties was a very fortunate action. The rapid development of fatigue testing and safety policies with regard to fatigue would probably have come rather soon even without ICAF, but it is obvious that the Committee under the direction of Plantema very soon got a remarkable influence on the fatigue research in a large part of the world by suggesting the urgent problems to be studied and joining the efforts of many laboratories in different countries. The success of ICAF in the difficult beginning was not only due to the great skill and knowledge of Plantema and his collaborators but also to his diplomatic talents. Plantema was able to lead and unite people of very independent thinking, without ever raising his voice.

ICAF was benefitting from Plantema's friendly direction for about 15 years until his sudden death in 1966. We owe him many obligations for the platform he established, from which we can enjoy a fruitful international team-work. Especially in a small country like Sweden we feel that ICAF has meant a great deal for stimulating our efforts in a sensible direction to make the best use of our limited resources, and will continue to do so in the foreseeable future.

1. INTRODUCTION

Already some 25 years ago it was accepted among foreseeing people in the aeronautical field that fatigue failures would soon become one of the most serious problems in the first place in the design of transport aircraft with very long service lives required in order to keep the depreciation rate per flight hour low. The accelerated fatigue testing both on small specimens and full-scale structures and the extensive measurements of flight loads which took place during the fifties, revealed that it would not be possible to prevent fatigue cracks in the entire airframe by applying conventional safety factors on life. These factors have to be very large due to the scatter experienced in fatigue testing. The assessment of a safe life, free from cracks, is feasible only for specific components of the aircraft structure, for which a considerable weight increase may be tolerated, or which may easily be exchanged during the service life.

In the major part of an aircraft the operators thus have to live with the risk of fatigue cracks being present during the service life. When the designers became aware of this fact they introduced the "fail-safe" concept, which seems to have implied initially a duplication of the vital members or, at least a statically redundant structure. The idea was that when the structure had suffered a member failure which could not remain unnoticed for long, the structure should still be able to carry the "fail-safe load" equal to the limit load or a certain percentage of it. Duplication of members in a system may be a good solution for electronic components, but it does not usually provide weight-saving in a structure. Fail-safe properties can also be achieved, however, by giving the structure slow crack propagation rate, high residual strength and good inspectability of fatigue sensitive areas. A newer and more adequate name for this quality is "damage tolerance". The fail-safe concept will then require that regular inspections be performed by the operator.

The length of the interval between the inspections should be determined with regard to the strength reduction rate. This is reflected by the following definition given by BARROIS[1] at the ICAF Symposium in London 1973:

"The structure or one of its elements is damage tolerant when damage that is scarcely but definitely detectable during an inspection may grow during the interval before the next inspection, and if its magnitude after this period is low enough to permit the damaged structure to remain capable of withstanding fail-safe loads".

A slightly different approach was described by BUNTIN[2] who stated: "The inspection interval is then determined as the growth interval to failure divided by an appropriate confidence factor", where the growth interval is the service time from a defined initial crack size to another defined critical crack size.

Furthermore, the inspection procedure is of importance for the assessment of the safety of a fail-safe structure as expressed by POE-LEYBOLD[3]: "To prevent such a failure through inspection, a suitable procedure must find at least those fatigue cracks that are long enough to grow to failure before the next inspection. In order to allow for a crack to be overlooked once and thus provide some margin of safety, a crack growth period equivalent to two inspection intervals was chosen for this study."

A fatigue failure which occurs in an aircraft structure, e.g. a wing panel, during the interval between two inspections is the result of a series of unfortunate events. First a crack is initiated in a fatigue-sensitive point due to repeated loadings, usually of moderate magnitude. These load sequences also cause a crack propagation which in turn produces a strength reduction. At the inspection preceding the interval under consideration the crack may have been overlooked. Finally, a high load due to a heavy gust, or a manoeuvre, may exceed the present residual strength of the structure with total collapse as result. In establishing fail-safe criteria one has to take all these events into account. This can be done in a deterministic analysis by applying a number of safety factors based on judgement by experienced designers. Since the various service times to crack initiation and propagation to detectable and critical lengths are subject to large scatter, a purely deterministic approach which assumes the worst - or almost the worst - conditions, normally tends to give a very conservative design.

The aim of safety codes for certification of structures is to reduce the number of undesired outcomes where same sort of damage occurs. It may not be necessary to agree on a specific figure for the risk of damage which could also be described as "small", or "extremely remote", There seems to exist a general acceptance, however, of the risk, or the corresponding reliability, as the only logic basis for comparing different structures with regard to safety. Risk and reliability are statistical quantities that can hardly be estimated without some sort of probabilistic analysis. This is especially true when a very low risk is required as in the case of a catastrophic failure, and a complex behaviour is studied, as fatigue in a fail-safe structure, where several random variables have to be combined. Statistical analysis could be used not only to build up a primary design method but also for evaluating the present risk level existing in different types of aircraft structures and for calibrating the safety factors now applied, in order to obtain a consistent level of safety.

Although there are strong arguments for carrying out statistical calculations with regard to fatigue failure there is hardly any evidence in the literature that such calculations have been carried out in the development of actual aircraft structures. The rather few papers published in recent years represent mainly an academic view of the problem. The fact that people from the design offices and airworthiness authorities have not yet been able to analyse their structures statistically, or do not rely on the figures they have arrived at in the analyses possibly performed, is probably not due to the mathematical difficulties involved in the first place. A thorough treatment will be complicated but modern computer technology has provided us with means to overcome such problems. The main reason is more likely the lack of statistical data to support the calculations. This deficiency will never be cured in an entirely satisfactory manner. It is believed, however, that with a proper understanding of the physical phenomenon of fatigue failure, statistical calculations will still be of great value, let alone that the absolute risk values obtained may be far away from the actual ones.

The present report describes the details of a procedure for analyzing the probability of failure due to fatigue in a damage tolerant structure which is inspected at regular intervals, and discusses the statistical information available for such an analysis. It also reports the results from fatigue tests on aluminium alloy panels, which were carried out in two series with about 20 specimens in each. The first series included flight-by-flight cycling until detection of the first crack and subsequent propagation to various crack lengths. The residual strength was finally determined. In the second test series the number of flights were recorded for the first, second and third cracks and a common "parent" distribution was evaluated which included 50 test points.

Direct use has been made of the experimental results obtained, by introducing into probabilistic calculations distribution functions for time to crack initiation and for crack length at detection as well as crack propagation and residual strength curves determined. The test specimens were chosen with the primary aim to reduce cost of manufacture and testing. They are meant, however, to represent bottom wing panels of a transport aircraft. The computed diagrams giving the risk of fatigue failure may therebe considered to show some realistic trends, but their main interest should lie in demonstrating a mixed analytical-experimental procedure which could be followed in the design stage of a new aircraft.

2. SYMBOLS

a_s	crack length visible on one side of countersunk rivet head
$F(r)$ $f(r)$	distribution function and frequency function of residual static margin r
F_d	probability of crack detection
$G(t_d)$	probability of failure of structure containing crack, at time t_d after crack initiation
H	number of load amplitudes exceeding s_a in one hour, failure rate
H_o	load spectrum parameter, $H_o = 0.2$ in numerical example
h	load spectrum parameter, h = 20 in numerical example
L	crack length from tip to tip, L_0, L_1, L_2 and L_9 are specific crack lengths used in Eqs (21) and (22)
m	number of equally fatigue-sensitive members or spots also parameter of Eq (21)
N	number of load cycles with constant amplitude

P, P_ν	probability of fatigue failure during service life, and during inspection interval ν, respectively
P_c, P_{c1}, P_{c2}, P_{c3}	probability of crack initiation, first crack initiation etc
p_c	frequency function of crack initiation
R	residual static margin, R_o is original static margin
r	normalized residual static margin R/R_o
S	total load or stress
S_m	mean load or stress in flight
S_a	load or stress amplitude
s_a	normalized load or stress amplitude S_a/R_o
T_c, T_{c1} etc	service time to crack initiation, number of flights, to first crack initiation etc
T_o	scale-parameter of Weibull-distribution
T_u	parameter of Weibull-distribution, lower limit
T_ν	service time to end of inspection interval ν
t	service time in number of flights, or hours
t_d	service time from crack initiation
α	shape-parameter of Weibull-distribution
μ_c	mean value of logarithm of time to crack initiation
σ_c	standard deviation of log T_c

3. SURVEY OF FATIGUE RELIABILITY PROBLEM IN AIRCRAFT STRUCTURES

Detailed reviews of the present state of the art regarding the assessment of safety with respect to fatigue failure have been made by BARROIS[1] and HOOKE[4], where the certification requirements of existing airworthiness regulations are also dealt with. The survey of the problem is therefore limited to a few notes necessary for the correct understanding of the following presentation of a statistical method of analysis.

Fatigue failure occurs when a load acting on a structure has a magnitude exceeding that of the residual strength of the damaged structure. The probability of fatigue failure is thus the probability that the structure is subjected to a load exceeding its strength. The same statement applies to the static ultimate failure of an undamaged structure, the only difference being that the fatigue failure takes place in a damaged structure with a reduced strength and under a lower load. In a general study of the risk of failure it is consequently convenient to treat static failure and fatigue failure simultaneously without separating the risks. It is preferred in the present report to leave out the static failure and compute only the probability of fatigue failure, which takes place in a structure with a crack detectable with available methods of inspection. The separation has been made because it is felt that it may be of interest to know the magnitude of the failure risk that is due to fatigue. It could also be advisable to attach a lower risk level to fatigue failure than to a static failure from an excusability argument, as advanced by LUNDBERG[5].

The "safe life" of a structure may be determined from realistic full-scale fatigue tests by dividing the mean life until crack detection, or possibly complete failure, by a safety factor. A statistical interpretation of this factor is quite easy, since it is known that the service life until crack initiation may be represented by a log-normal or a Weibull-distribution, at least down to moderately low probabilities. The testing will give an indication of the magnitude of the standard deviation, which is also known to some extent, at least for smaller commonly used test specimens. Very low probabilities such as 10^{-3} or 10^{-4} may only be verified nominally, since extrapolations of the distribution functions to these levels are beyond our present knowledge. The safety factors must be set at correspondingly large values. Where fail-safe methods of verification are allowed, they are usually preferred before the safe-life approach.

According to Hooke[4] "fail-safe methods are applicable to redundant structures and even to non-redundant ones, if, at each critical area, there exists a 'crack size detectable with certainty', if this is smaller than a 'critical crack size' at which the structure reaches an unsafe strength, and if the rate of propagation of a crack from the first to the second is known". In the conventional deterministic methods used,the information required may be summarized in two points:

1. Critical crack length, where the residual strength may just carry the fail-safe load, equal to the limit load or of similar magnitude.
2. Crack propagation time from a detectable length to critical length. The detectable crack length is determined with regard to the inspection technique to be used.

The inspection interval is determined by dividing the crack propagation time by a factor of safety. This method obviously disregards the risk of meeting a higher load than the fail-safe load during the inspection interval. On the other hand it assumes that a crack is always present at the beginning of the interval which implies an overestimation of the risk. Since inspections are expensive for the operator the designer aims at long crack propagation times. This can be achieved by increasing the crack propagation life, either by improving the fracture toughness of the structure, which gives a lower crack propagation rate and a longer critical crack, or by employing more refined inspection techniques, see VLIEGER[6]. The conventional fail-safe method does not seem to provide a low probability of failure, however, if the inspection intervals are increased to a magnitude of 10000 h, since the risk of encountering a load exceeding the limit load will then become considerable. BARROIS[1] also arrived at the conclusion that the fail-safe verification does not always result in a sufficiently high level of safety:

"It is more advisable to define areas to be inspected and the periodicities of maintenance inspections capable of detecting cracks of moderate length than to accept that only one or two inspections be made during the whole specified life of a particular aircraft, and to compensate for this concession by requiring manufacturers to consider cracks of such lengths that an error or scatter relating to their critical sizes or to the fatigue strength of reinforcements aimed to delay the crack propagation, could make fail-safe illusory and entail a catastrophic failure".

A cyclic sampling inspection procedure does not improve the safety to a notable extent since the important parameter is the inspection intervals of the individual aircraft as found by EGGWERTZ-LINDSJÖ[7] and STAGG[8].

A complete analysis of the fail-safe system should include the following five quantities

1. Service life until crack initiation under random loading
2. Crack propagation rate for the same loading
3. Residual strength at given crack length
4. Length of crack detected at inspection
5. Maximum load applied during inspection interval

The information required concerning the statistical distribution of the crack initiation life time is not at all so extensive as for the safe-life verification described, since it is not usually necessary to employ lower probabilities than about 1 per cent. A study of the list reveals that quantities Nos. 1, 4 and 5 are typically stochastic variables with so large scatter that "safe" assumptions seem rather meaningless. Using fracture mechanics theory it is possible to compute the crack propagation rate as well as the variation of residual strength with the crack length. With test results available from realistic specimens it should be quite easy to choose conservative values considering the small scatter. The ultimate strength of an undamaged structure is known to be a stochastic variable with a moderate coefficient of variation for metallic materials. The same should obviously apply to a structure with a crack of given length. This variation could probably be covered by introducing instead of the median value a characteristic value corresponding to a low percentile. A better procedure is to use a rather well-established distribution function for the residual strength.

It may be feasible to prepare a method for analysis of fail-safe structures which is more elaborate than the present conventional method by employing a number of partial safety factors. In order to attain a consistent standard of safety it would be necessary to perform numerical calculations with a statistical procedure for a large number of design cases in order to calibrate the safety factors. The most complete statistical method so far has been presented by YANG-TRAPP[9], see also HOOKE[4,10] and EGGWERTZ[11].

Even if the information concerning the five quantities listed above, is the most adequate and complete one could hope to collect, the life evaluation by a rather accurate statistical method would still deviate considerably from observed actual service lives on the average. The reasons for poor correlation have been discussed by LOWNDES[12] and TROUGHTON-WOODFORD-HARPUR[13]. Among more important factors are difficulties to apply a correct loading system on the test specimens and to simulate a corrosive environment equivalent to service conditions. Extra safety factors or introducing coefficients of variation of "engineering judgement" will be necessary. Finally, the unexpected failures, "inconceivable events", resulting from grave errors in design, manufacture or operation, or pure accidents, can never be brought into a normal design analysis. Damage tolerance and frequent inspections may, however, prevent incidents to develop into catastrophic collapse.

4. STATISTICAL ANALYSIS OF ENTIRE COURSE OF FATIGUE FAILURE AT REGULAR INSPECTIONS

A fatigue failure occurs in a structure where a crack has first been initiated, leading to continuous strength reduction, when the structure afterwards is subjected to a random load the magnitude of which exceeds the residual strength at that moment. Within the first inspection interval the probability of fatigue failure is equal to the joint probability that a crack is initiated during a time increment dt and that the structure afterwards meets a load exceeding its strength integrated over the whole interval T_1. The probability of crack initiation is $p_c(t)dt$ and the conditional probability of strength exceedance is denoted $G(T_1 - t)$. The probability of failure P_1 is thus

$$P_1 = \int_0^{T_1} G(T_1 - t)p_c(t)dt \tag{1}$$

where $p_c(t) = dP_c(t)/dt$

The distribution function for service life to crack initiation caused by a random loading spectrum, may either be assumed to be log-normal

$$P_c(t) = \frac{\log e}{\sigma_c \sqrt{2\pi}} \int_0^t \exp\left[-(\log x - \mu_c)^2/2\,\sigma_c^{\,2}\right]\frac{dx}{x} \tag{2}$$

or a Weibull-distribution

$$P_c(t) = 1 - \exp\left\{-\left[(t - T_u)/T_o\right]^{\alpha}\right\} \qquad (3)$$

where

$\mu_c = (\overline{^1\log}\ t)$ and σ_c are the mean value and standard deviation of Briggs's logarithm of the service time until crack initiation.

T_u, T_o and α are the lower limit, the scale- and the shape-parameters of the Weibull-distribution.

The probability of failure in a damaged structure is a function of the service time from crack initiation to the end of the inspection interval $t_d = T_1 - t$ and is written $G(t_d)$. This function is determined by the random loading spectrum and the residual strength decrease function.

The load S on the structure is assumed to consist of a mean load S_m which is constant during the whole service life, and an amplitude S_a which varies randomly

$$S = S_m + S_a \qquad (4)$$

This simplified assumption is only made for the loads causing ultimate failure, while the variation of mean load in the ground-air-ground cycles obviously should be taken into account when determining the fatigue life parameters of Eqs (2) and (3).

Extensive measurements of gust loadings on different types of aircraft have shown that the following exponential load exceedance distribution represents a satisfactory approximation [14, 15].

$$H(S_a) = H_{01}\ \exp(-h_1 S_a) + H_{02}\ \exp(-h_2 S_a) \qquad (5)$$

The first term corresponds to the contribution from clear-air turbulence which is of dominating importance for the fatigue crack initiation and crack propagation. The second term gives the heavy gusts in extreme turbulence. These are responsible for the ultimate failures in structures with or without fatigue cracks. In a fail-safe system with comparatively short inspection intervals implying that only moderate reduction of the ultimate strength is allowed, the first term may be omitted. Introducing the normalized load amplitude

$$s_a = S_a/(U_o - S_m) = S_a/R_o \qquad (6)$$

the number of load amplitudes exceeding s_a in one hour may be written

$$H(s_a) = H_o \exp(-hs_a) \qquad (7)$$

U_o is ultimate strength of the undamaged structure. The load amplitude is thus normalized with respect to the original static margin R_o. The two parameters H_o and h of the extreme gust load spectrum may be obtained from published turbulence recordings for the type of aircraft and operation under consideration.

The residual strength U of the structure decreases with the time t_d after crack initiation. The residual static margin R normalized with respect to R_o may be written.

$$r(t_d) = R/R_o = (U - S_m)/R_o \qquad (8)$$

It is also assumed that the static margin $r(t_d)$ has a distribution function F(r) and a frequency function f(r), where either a normal or a log-normal distribution may be introduced. The mean value and standard deviation of $r(t_d)$ may be determined from fatigue tests on representative specimens where the crack propagation is recorded and the residual strength is also determined for various crack lengths. If the number of test specimens is small, the standard deviation may be estimated from earlier tests on similar structures.

An alternative method to determine the function $r(t_d)$ would be to use formulae deduced in fracture mechanics to determine first the crack propagation rate under the random loading which has been adopted also in the evaluation of the crack initiation parameters. The increase in crack length L with service time t_d from crack initiation is then obtained by integration. The residual strength U can also be computed by fracture mechanics as a function of L. Finally the normalized residual static margin $r(t_d)$ is found according to Eq (8).

Fatigue failure is triggered by a load amplitude exceeding the residual static margin at the moment this load is met

$$s_a \geq r(t_d) \qquad (9)$$

The number of failures in one hour, also called the failure rate, is obtained by inserting Eq (9) into (7)

$$H(t_d) = H_o \int_o^{\infty} f(r)\ \exp(-h\bar{r}_d)\ dr(t_d) \qquad (10)$$

where f(r) is the frequency function of r and $\bar{r}_d$ is the corresponding mean value. Assuming no correlation between the load amplitudes the probability of failure of a damaged structure is now[11]

$$G(t_d) = 1 - \exp(\int_o^{t_d} H(t_d)\ dt_d) =$$

$$= 1 - \exp \left\{ -H_o \int_o^{t_d} \left[\int_o^{\infty} f(r)\ \exp(-h\bar{r}_d)\ dr(t_d) \right] dt_d \right\} \qquad (11)$$

At the end of the first inspection interval, $t = T_1$, inspection is performed using a specified procedure. Normally a longer crack will be easier to detect than a shorter one. Rather few special investigations have been made regarding the reliability of the various NDI techniques available. It should be possible to evaluate the crack lengths detected in the numerous laboratory fatigue tests with large sample sizes which have been run so far. This would give distribution curves for detectable crack length under laboratory conditions. Service inspection cannot usually attain the same accuracy as in laboratory for economical reasons. One will probably have to specify that the probability of detection must exceed given percentiles at two different crack lengths, which will determine the inspection method required.

It is presumed that a continuous distribution curve is available which gives the probability that a crack of a given length L is detected. If the relationship between L and t_d is known, as discussed above, and deterministic, the distribution function for the time t_d from crack initiation to crack detection will be easily found and denoted by $F_d(t_d)$. There is obviously a scatter in time to produce a certain crack length but this scatter is rather likely to be unimportant compared to the crack detection time.

After the first inspection, i.e. at the beginning of the second inspection interval the following cases may be distinguished.

I. No crack has been initiated

II. A crack has started

a. The crack was not discovered at the first inspection

b. The crack was found and repaired at the inspection

III. Complete fatigue failure has occurred

Case I applies to $\left[1 - P_c\ (T_1)\right]$ of all structures in a thought population. No change of initial conditions has taken place and Eq(1) is still valid for the second interval if T_2 is introduced instead of T_1 and the integration is carried out between the limits T_1 and T_2

$$P_2^{\ I} = \left[1 - P_c(T_1)\right] \int_{T_1}^{T_2} G(T_2 - t)\ p_c(t)dt \qquad (12)$$

Case III applies to a percentile P_1 which is supposed to be a very small value. It is therefore reasonable to neglect P_1 compared to one and assume that Case II has a share $P_c(T_1)$.

Case II a will then be valid for a portion

$$\int_o^{T_1} \left[1 - F_d(T_1 - t)\right] p_c(t)dt \text{ and II b for}$$

$$\int_o^{T_1} F_d(T_1 - t)\ p_c(t)dt$$

Case II a will thus yield the following contribution to the probability of failure during the second inspection interval

$$P_2^{\ IIa} = \int_o^{T_1} \left[1 - F_d(T_1 - t)\right] p_c((t) \left[G(T_2 - t - G(T_1 - t)\right] dt \qquad (13)$$

It is postulated first in Case II b that the repair is carried out in such a way that the structure is equivalent to one without crack but of the same fatigue age T_1. This is probably a conservative but rather realistic assumption for a built-up structure including a large number of fatigue-sensitive points of which only one has been renewed. This subcase gives the probability

$$P_2^{II\ b1} = \left[\int_0^{T_1} F_d\ (T_1 - t)\ p_c(t)\ dt\right]\left[\int_{T_1}^{T_2} G(T_2-t)\ p_c(t)dt\right] \qquad (14)$$

Another supposition which is likely to be unconservative but rather realistic for a structure with only one critical point is to treat a repaired structure as completely renewed with fatigue age $t-T_1$. From this subcase is obtained

$$P_2^{II\ b2} = \left[\int_0^{T_1} F_d\ (T_1 - t\ )\ \ p_c(t)dt\right]\left[\int_{t_1}^{T_2} G(T_2 - t)p_c(t-T_1)dt\right] \qquad (15)$$

The probability of failure during the second inspection is the sum

$$P_2 = P_2^{1} + P_2^{II\ a} + \begin{cases} P_2^{II\ b1} \\ \\ P_2^{II\ b2} \end{cases} \qquad (16)$$

By determing the risk that a crack is not detected and the probability that it is found and repaired at the second inspection, $t = T_2$, it is possible to proceed to the next inspection interval, where the number of cases will be further branched off. It is possible, however, to build up recurrence formulae suitable as a basis for a computer programme.

In case all cracks are detected at the inspections, i.e. $F_d(t_d) = 1$, and the first subcase of II b is postulated, it is seen from Eqs (12) and (14) that the probability of failure during the second or any other inspection interval ν may be written

$$P_\nu = \int_{T_{\nu-1}}^{T_\nu} G(T_\nu - t)p_c(t)\ dt \qquad (17)$$

When the probabilities of fatigue failure for all inspection intervals are small, as they should be in a safe structure, the probability of failure during the whole service life of n intervals is obtained as the sum of the probabilities for the intervals

$$P = \sum_{\nu=1}^{n} P_\nu \qquad (18)$$

The analysis presented does not require the lengths of the inspection intervals to be equal or any other periodicity, but constant inspection intervals are usually preferred for many reasons.

5. INFORMATION AVAILABLE TO SUPPORT ANALYSIS

A common argument advanced against introducing probabilistic methods in the design of structures has been the undeniable fact that statistical information is not available to such an extent that conclusions with a high confidence can be drawn from such analyses. This is true - and will remain so for a long time - not only concerning the probabilistic procedures but often still more for conventional methods with one or several safety factors.The probabilistic analysis makes a better use of the information in the form of measured data, even if it is meagre. Sometimes it may be difficult to find the adequate information because of lack of understanding of the physical phenomenon to be treated, but this problem can be overcome just by statistical thinking.

The aim of this chapter is not to present a collection of all pertinent statistical information available in literature but rather to give examples and discuss the usefulness of methods to generate information.

Loadings

The present study is mainly devoted to wing structures, which are subjected to loads caused by wind gusts and manoeuvres. This is true also for the tail unit, while the dominating loads on the fuselage come from the pressurizations. Statistical data of accelerations measured near the centre of gravity of the airplanes and simultaneously recorded values of velocity and altitude have been collected by research establishments such as NASA[14] formerly NACA, and RAE[16] with its predecessor for at least 3-4 decades. These data can be converted into gust velocities and finally, by introducing the gust response of the airplane structure, into loads and stresses in the wing area under consideration. The gusts and to a large extent also the manoeuvres are random occurrences. The gust loadings have been analyzed since the fifties as a stationary Gaussian random process, see PRESS-MEADOWS-HADLOCK[17] and BUXBAUM-ZASCHEL[18], with a single parameter, the root-mean-square value. The analysis gives a statistical distribution for the loading, also called a load spectrum. The gust recordings have been made on different routes under different weather conditions and at different altitudes. It has been found that although the severity of the gusts varies with the conditions, the shape of the distribution curves is approximately the same in all cases. For engineering purposes a number of simplified distribution functions have been proposed e.g. by BULLEN[19], BUXBAUM-SVENSON[20] (log-normal) and HOBLIT-PAUL-SHELTON-ASHFORD[21] (two exponential distributions).

The random loadings have to be taken into account as responsible for first the crack initiation and then the crack propagation. Finally a heavy load causes the ultimate failure. It has been found that fatigue crack initiation and propagation are considerably influenced by the sequence in which the loads are applied. As concluded by SCHIJVE[22] the linear damage theory by PALMGREN-MINER should therefore normally be avoided. Since there is no other theory available that gives generally more reliable results, the only recommendable way to obtain information regarding crack initiation and crack propagation is to carry out flight by flight testing where the loadings are simulated as realistically as possible. Every flight begins with a number of taxiing load amplitudes around a mean load corresponding to ground condition. The mean load is then changed to 1-g flight condition where a large number of gust or manoeuvre amplitudes are applied in a random sequence. In order to reduce testing time, one is usually inclined to omit smaller amplitudes which occur in large numbers according to the load exceedance distribution. Even amplitudes amounting only to a few per cent of the ultimate load, however, may have a considerable influence on the crack propagation. Truncation of high loads provides another difficult judgement. High load amplitudes cause residual stresses which are often beneficient with respect to fatigue. Single high loads may stop the crack propagation for a considerable time. For specific recommendations reference is made to SCHIJVE[22].

Fatigue testing has been performed in the past mainly with a constant mean load and constant amplitudes. GASSNER[23] introduced the programme testing, where the load amplitudes were varied a number of times forming blocks with a composition of the amplitudes in accordance with the applicable load spectrum. This procedure was followed by random testing[24], which was made possible with the development of the servo-hydraulic testing machines. Finally the computerized equipment now available in modern laboratories, allows flight simulation testing where a large number of flights of different severity can be randomly introduced. The variations in load sequences that can be achieved is thus practically unlimited. This situation might soon lead to increased difficulties to compare test results which have been obtained with some differences in loading. The German Laboratorium für Betriebsfestigkeit LBF and the Dutch Nationaal Lucht- en Ruimtevaartlaboratorium NLR have therefore developed together a flight by flight test programme for the fatigue critical wing root area, which will be used by the two laboratories as a standard flight simulation programme[25]. Very likely, it will also be extensively employed by laboratories in the other ICAF countries. The standardization is probably necessary to form a basis for future international agreement on certification requirements. The load spectrum chosen has been truncated in the upper and lower ends in order to provide a conservative mean fatigue life according to present knowledge. Also the scatter of the fatigue lives is influenced by truncation of high and low loads and their sequences, and it is not quite obvious that the standardized load programme will provide conservative values of the standard deviation of the fatigue life. For this reason one should hope that future fatigue testing for research purpose will partly be carried out with realistic loading programmes which differ from the standard, until we are able to establish a general theory for fatigue life analysis.

The ultimate failure of a structure having a fatigue crack, may, from theoretical point of view, take place at any moment from the crack initiation until the residual static margin according to Eq (8) has decreased to zero. In order to keep the probability of failure low it is necessary to inspect the structure at intervals which are only a fraction of the entire crack propagation time. Long cracks and small residual static margins will consequently never appear in a structure of adequate strength. Only relatively high loads are thus of interest. The distribution function of heavy loads caused by storm turbulence or manoeuvres cannot reliably be determined by extrapolating from records of every load amplitude measured during a representative service time. A much better procedure is to include only the maximum values met per flight or even per week or month, and possibly per year, where the service time required for measurements gets quite extensive, however. Extreme value distributions, discussed in detail by GUMBEL[26,20] are likely to fit the recorded values quite well. Other distribution functions have also been found to be suitable, such as log-normal or exponential distributions. The latter has been preferred in this report, see Eq (5), since parameter values are available in the literature for a large number of flight conditions[15,27,28].

It has been observed by BULLEN[19] that the correlation between successive flights with regard to magnitude and number of gusts is very low. Within a flight, on the other hand, the gust loads, as might be expected, are not independent. The assumption of independence made in the analysis in Chapter 4 is considered to be conservative[9,29].

Service life until fatigue crack initiation

Fatigue life determined in constant amplitude tests have mostly been expressed in number of cycles N. With the introduction of flight simulation testing the natural unit is number of flights, or possibly hours of flight if each flight comprises many hours. The word time, denoted by t, or life time, will still be used in this report for the life.

An estimate of the service time until crack initiation for preliminary design can usually be made by comparison with earlier airframes where test results and service experience are available. For the final design and for certification work one has to resort to fatigue testing. In the analysis of the life times to crack initiation obtained in the tests, it is assumed that they follow a certain distribution function. The log-normal distribution, Eq (2) has been widely used since long ago. Recent investigations by STAGG[30,31] indicate that this is the two-parameter distribution that normally gives the best fit to a limited number of test points. Also the WEIBULL-distribution[32] was introduced at an early stage and has been frequently applied. In its general version, Eq (3), it includes three parameters, which makes it easy to adapt even for larger sample sizes. The evaluation of the three parameters complicates the use of this distribution. The lower limit T_u has therefore in most cases been neglected, yielding a two-parameter distribution, often simply called extreme value distribution

$$P_c(t) = 1 - \exp\left[-(t/T_o)^{\alpha}\right] \qquad (19)$$

The Weibull-distribution, contrary to the log-normal distribution, exhibits an increasing failure rate with time when $\alpha > 1$, which is in agreement with the physical model. Regarding the distribution functions mainly as mathematical approximations they seem to have quite similar appearance for $T_u = 0$ and $\alpha \searrow 2$.

The mean value of log t may easily be determined with the confidence required from a limited number of test results. In order to obtain the standard deviation of log t or log N one has to rely on earlier experience from large sample sizes. Published investigations, which are mainly referred to constant amplitude testing, seem to indicate values between 0.1 and 0.3, where long lives usually correspond to high standard deviations[33,34,35]. For normal service lives especially in flight simulation testing on sheet material specimens a value of the logarithmic standard deviation $\sigma_c = 0.2$ may be considered conservative. Very few fatigue tests have included more then 100 specimens and these have been quite small and often unnotched. An interesting study has been made at the Boeing Company [36] in an attempt to collect larger sample sizes. More than 2000 groups of data representing approximately 11,000 specimens of aluminium alloys were analyzed. Results from similar test groups were pooled together giving consistent results which confirm our earlier knowledge on scatter.

In the Weibull-distribution the scale-parameter T_o, also called the characteristic value, corresponds to the mean value of the log-normal distribution, although it is somewhat larger, yielding a probability $1 - 1/e = 63$ per cent. The shape-parameter α may be interpreted as a scatter-parameter which is correlated to the standard deviation of log t. A value of $\sigma_c = 0.14$ seems to correspond to $\alpha \approx 4$, while the conservative value $\sigma_c = 0.2$ would give $\alpha \approx 2.5$ in the two-parameter Weibull-distribution Eq (19). The relations seem to change considerably in the three-parameter Eq (3).

The values of the scatter-parameters mentioned refer to variations in material properties, configuration and manufacture of the specimens only, since the variations in loading between different tests are likely to be insignificant in modern fatigue testing machines. For the airframe structure in service there is a certain random variation of the loading between airplanes operated similarly on the same routes. These variations can be measured in flight and taken into account by introduction of the variance of the load spectrum, which should be added to the variance of the fatigue test life. In the future it may become possible to include in a computer controlled testing machine a realistic random variation of the flight load simulations from specimen to specimen, thus including the entire scatter in the test results. There remains some problems to solve, as indicated in the discussion of the loadings, before such a procedure can be considered reliable.

Crack propagation rate

The crack initiation may be said to occur at the moment when the crack, which has existed earlier as a micro-crack, can be observed using e.g. the dye penetrant method. The crack propagation, starting at the crack initiation, is thus a macro-crack growth from a crack length L_o just detectable.

Research on crack propagation was accelerated in the middle of the fifties when the introduction of the fail-safe concept made it necessary to be able to anticipate the length of the crack propagation period, see SCHIJVE[38]. It was found that the growth of a crack per load cycle depends on a number of factors among which the stress level, especially the stress amplitude, the type of stress concentration and the material properties are supposed to be of primary importance. Empirical formulae were established and theoretical approaches were also made. The stress intensity factor K introduced in fracture mechanics has turned out to be a useful tool for studying the crack propagation rate. The law originally introduced by PARIS[39]

$$dL/dN = C\ (\Delta K)^n \qquad (20)$$

has been verified for constant amplitude load cycling, where ΔK is the range of the stress intensity between maximum and minimum values, C and n being empirical constants. Extensive testing both of unstiffened sheets and stiffened panels of aluminium may be quoted [40, 41, 42]. Also crack propagation under stationary Gaussian random loading can be correlated with ΔK if the stress intensity factor is modified to include a measure S_{rms} of the rise and fall of the Gaussian process.[39,43]

It has not been possible, so far, to extend the application of Eq (20) to flight simulation loading which would require a generally valid cumulative crack propagation theory[44]. As noted in the discussion of the loadings, such a theory does not seem to be feasible at the present state of knowledge. A relationship between crack length and service time t_d from crack initiation, obtained by integration of Eq (20) in some modified version, could possibly be useful at the preliminary design stage. For more accurate design calculations and verification one can rely only on crack propagation tests carried out under realistic conditions, where the difficulties met in assessing the appropriate loads are still worse than for the crack initiation testing.

Macro-crack propagation testing shows low scatter for all types of loading compared to the scatter present in crack initiation as long as the loadings are not varied between the different test specimens[22]. The crack length could approximately be regarded as a deterministic function of t_d provided the possible variations in loading are taken care of separately.

Residual strength

The original ultimate strength of a structure is reduced by the presence of a crack, The length L of the crack is obviouly a dominating parameter for determining the residual strength. In a plain metal sheet with a central crack it is possible to determine with good accuracy the residual strength as a function of the crack length using the linear fracture mechanics theory, provided the critical stress intensity factor K_c of the sheet material is known[45]. By use of finite element methods for computing the stress concentration at the crack tips it is also possible to treat stiffened panels as shown by VLIEGER[6] and POE[46]. The main advantage inherent in the presently growing ability to compute the residual strength of even rather complicated structural parts is that it provides opportunity to analyze a large number of configurations at an early design stage in order to find an optimal solution. This is obviously not possible if one has to rely only on testing, even if residual strength tests are not as expensive as fatigue tests. A large number of residual static strength results from wing panels, box beams and other structures have been published during the fifties and sixties. A review of many investigations was made by BRANGER[47].

The residual strength U like the ultimate static strength U_o is subject to scatter. In the analysis of the probability of static failure it is important to take this scatter into account as shown by FREUDENTHAL - PAYNE[48]. The reliability calculations concerning fatigue failure of fail-safe structures include a number of quantities which are not yet known with a fair accuracy. This makes it less urgent to include the rather small scatter of the residual strength. YANG-TRAPP[9] observed that the resulting failure rate would be very unconservative without treating U as a random variable. This is obviously true if the mean value of U is introduced, but not necessarily so when a characteristic value corresponding to a lower percentile is used instead, which is a common procedure.

Information about scatter of residual strength seems to be scarce in the literature. One could probably derive results of interest from the scatter of the critical stress intensity factor. It is likely to be conservative, however, to assume that the coefficient of variation of U is the same as for the orginal strength U which is normally around or slightly above 5 per cent. It should be an acceptable approximation to use a Gaussian distribution function for the ultimate static strength, but a log-normal or a Weibull-distribution usually give a better fit to test points in the lower tail [49,50]

Crack detection at inspection

Nondestructive testing NDT and nondestructive inspection NDI which is a technique used to locate, identify and measure fatigue cracks, has been developed rapidly in recent years with the introduction of the fail-safe concept and the advent of the modern fracture mechanics.

It is now possible to introduce quantitative NDI criteria in design specifications, and this has been done in same engineering fields. A large variety of inspection techniques are used such as X-ray, magnetic particle, dye penetrant, ultrasonic and eddy current methods. People who are regularly carrying out inspections in service or in the laboratory would know quite well which crack lengths they are usually able to find with and without equipment. Such information has rarely been published, so far. A few examples will be quoted from the ICAF Symposium in London 1973. STONE[51] made the following statement when describing the fail-safe requirements for the DC-10:

"In a walkaround inspection there is a 50-percent chance of finding a 5- to 6-inch-long crack. A scheduled inspection has a 50-percent chance of finding a 1- to 3-inch crack. The use of a visual aid, such as a magnifying glass, can show up a 0.5 inch crack."

BARROIS[1] gave a somewhat more optimistic view:

"On the visible external skin of aircraft, French Airline Companies have discovered no cracks longer than 50 mm (2 inches). Larger cracks have been found in hidden or masked areas." ....
"As an order of magnitude, apparent length of detectable cracks might be:

L = 10 mm for visual inspection after local cleaning
5 mm for inspections using a magnifying glass and for X-ray inspections
1 mm for reamed fastener holes using an eddy current sensor"

There should be much material available from laboratory fatigue testing of large sample sizes, where the crack lengths have been recorded at detection. The first systematic investigation more widely known was published in 1969 by PACKMAN-PEARSON-OWENS-YOUNG[52]. The test specimens used were seamless tubes with 76 mm diameter and 6 mm wall thickness made of 7075 aluminium alloy or 4330 vanadium modified steel, employing X-ray, magnetic particle, dye penetrant, ultrasonic and eddy current inspection. The percent of cracks found F_d, called "sensitivity" by the authors, was plotted versus the crack length for the various inspection methods. Fig 1 shows the distribution curves determined for 7075 aluminium with ultrasonics, dye penetrant and X-rays, where both axes have linear scales.

The empirical investigation by PACKMAN et al has been discussed by a number of authors some of which have proposed distribution functions. YANG–TRAPP[9] gave the following relationship

$$F_d\ (L) = \left[(L - L_1)/(L_2 - L_1)\right]^m \qquad (21)$$

within the interval $L_1 \leq L \leq L_2$

where L_1 is the minimum crack length below which the crack cannot be detected and L_2 is the maximum crack length beyond which it can be detected with certainty, while m is a parameter to be determined empirically. By introducing $L_1 = 0.5$ mm, $L_2 = 7.5$ mm and $m = 1$ a straight line is obtained in Fig 1 which approximates quite well the curve given for the dye penetrant method.

A more theoretical approach was made by DAVIDSON[53] who proposed the detection function

$$\begin{aligned} F_d(L) &= 0 && \text{for} \quad L \leq L_o \\ F_d(L) &= C_1 \left\{1 - \exp\left[-\beta_1 (L - L_o)\right]\right\} && \text{for} \quad L \geq L_o \end{aligned} \qquad (22)$$

L_o is the threshold value of detection and C_1 is a value slightly below one, indicating that the probability of crack detection is never 100 per cent even for very long cracks. The parameter β_1 is meant to be determined by a critical crack criterion stating that the probability of detecting a critical crack of length L_c should exceed a fixed value. Introducing $L_o = 1$ mm and $C_1 = 0.98$, requiring that 0.9 of all cracks of length $L_c = 4$ mm be detected gives the equation

$$F_d\ (L) = 0.98 \left\{1 - \exp\left[-0.835\ (L - 1)\right]\right\} \qquad (22a)$$

which has also been introduced in Fig 1. This curve seems to be a reasonable approximation in the lower part for ultrasonic inspection.

Further results from laboratory investigations have been published by KNORR[54].

When dealing with crack detection in service inspection where the crack length variation is much larger, it seems advisable to introduce a logarithmic scale for L, whereas the probability could be Gaussian. Fig 2 is such a log-normal diagram where Eq (21) has been plotted with the parameters L_1 = 0.5 mm, L_2 = 50 mm and m = 0.2, used by YANG-TRAPP[9] in a numerical example including service inspection. The laboratory detection curves according to YANG-TRAPP and DAVIDSON[53] shown in Fig 1, have also been drawn for comparison. The statement by STONE[51] concerning scheduled and walkaround inspections has been represented by scatter lines at the 50 per cent level.

It is hoped that the crack inspectors working at the operators or manufacturers of aircraft, will soon provide us with more data on detection lengths.

6. PANEL TESTS AT FFA

The fatigue literature contains abundant information concerning loadings, crack initiation life, crack propagation and residual strength of wing panels and rather little on crack detection, as may be concluded from Chapter 5. Although almost all fatigue testing which has been reported in detail will be able to provide material for a probabilistic analysis, there is still a need for more experimental results especially from larger sample sizes with nominally equal test specimens. Parallel to theoretical studies aiming at methods for determining safe inspection intervals, two series of similar test panels have been fatigue cycled simulating repeated flights of a transport aircraft on routes of medium lengths. The investigations have been described in two earlier reports[11,55] and are only summarized here.

Test specimens and test procedure

The fatigue test panels were manufactured according to Figs 3 and 4 in the first and second test series respectively. Within the test length it consists of a 2.5 mm sheet of 2024-T3 aluminium, clad and anodized (SAAB-SCANIA specification 3526-68), with a width of 480 mm. The sheet was provided with four parallel rows of small strips of the same sheet material, four in each row. The strips, 38 x 20 mm, were fastened to the panel by two 5 mm aluminium rivets, countersunk on the free side of the sheet. The total number of rivet holes was 2 x 4 x 4 = 32. The load transfer to the four strips in the same cross section was measured to be about 3 per cent of the total tensile load on the specimens. In order to minimize bending in the sheet the two outer rows of strips were attached to one side and the inner rows to the other side of the panel.

The panels and the small strips of the first test series were cut from four sheets, 1000 x 3600 mm, of the same delivery, while the specimens of the second series were taken from ten sheets, 1000 x 1200 mm, of a new delivery some four years later. Three material coupons in the longitudinal direction of the sheets were tested in tension from each series. The mean values of the ultimate stress, 0.2 yield stress and modulus of elasticity were 458, 350 and 69800 MN/m^2 respectively in the first series. The second series gave very similar values, 459, 337 and 69700 MN/m^2.

After fatigue testing of the first series, the damaged panels and two panels without cracks were loaded statically in tension to determine their residual strength. Since the intention was to obtain experimental information for a realistic stringer-stiffened wing panel, the strips were removed from the sheets before the static tests and replaced by stringers continuous along the whole length of the panel as shown in Fig 5. The stringers were made of channel section bars, 41 x 36 x 2.7 mm, of extruded 2024-T4 aluminium (SAAB-SCANIA specification 3526-4).The total stringer area, 4 x 300 = 1200 mm^2, was equal to the cross-sectional area of the main sheet.

The panels of the second series, which were only fatigue cycled, differed slightly in the grip areas from those tested in the first series, as may be seen in Figs 3 and 4. The doublers were extended along the edges, forming U-shaped plates, while the earlier specimens had rectangular doublers. It was found by strain gauge measurements that the rectangular doublers gave a stress which was about 5 per cent higher at the inner rows of rivet holes than at the outer rows. The first cracks also occurred more frequently in the inner rows. This difference was considered unsatisfactory since the aim was to treat all the stress concentration spots statistically as being identical. Finite element calculations were carried out for a number of doubler configurations using the MRI STARDYNE programme. These showed a difference in stress between the inner and outer rows decreasing from about 7 per cent for rectangular doublers to 3 per cent for the configuration chosen. Strain guage measurements on the new test panels also showed about 3 per cent difference. An earlier investigation by WHITTAKER-GERHARZ[56] has indicated that this improvement is what one can reasonably expect.

The fatigue cycling was performed in a servo-hydraulically controlled testing machine with a maximum loading capacity of ± 0.20 MN, shown in Fig 6. For the first ten specimens of the first test series the original controlling and programming units of the machine were utilized, which automatically provided a repeated load programme including eight steps. Since these units caused some trouble, they were replaced in the rest of the investigation by a punched-tape-operated programme unit, Fig 7, and a home-built control unit. The only nominal difference in the loading was that the latter system had to be run at a lower speed, 70 per cent of the speed of the original system.

Fig 8 shows the stress amplitude distribution curve used, which is typical for a jet transport with a flight duration of 1 h. The test programme included six different stress amplitudes ranging from 30 down to about 10 MN/m^2. No higher loads were thus included than those which are normally encountered in every flight. The flight simulation programme is described in Fig 9. It starts with the ground condition with compression $S_{min} = -25$ MN/m^2, all taxiing loads being omitted. The mean stress is then changed to tension corresponding to flight condition, $S_m = 88$ MN/m^2, and six steps with the amplitudes 10.3, 15, 24, 30, 19 and 12.5 MN/m^2, are cycled around this mean stress. The programme includes 88 load cycles besides the change in mean stress from ground to flight condition, and was completed in 4.35 s in the first ten tests and then in 6 s. Until the initiation of the first crack, which was recorded automatically by crack wire, the machine could be run continously 24 h per day. The second and third cracks were detected by frequent inspections, using also the crack wire where it could easily be repaired.

In the first test series panel No. 1 was fatigue cycled until complete failure, while the other panels were cycled until they developed at least one crack of a length varying between 10.5 and 91 mm. Altogether 18 records of the number of flights to the initiation of the first crack were obtained. One of the specimens was subjected to a somewhat lower stress level than the others, leaving a sample size of 17 equivalent tests.

After the fatigue panels had been modified as described above, the clamping jaws used in the fatigue machine, were installed in a 5.0 MN uniaxial testing machine. In this machine 17 panels with cracks of various lengths and two panels without cracks were subjected to increasing tension load to ultimate failure. One of the panels without cracks had not been fatigue tested while the other had withstood more than 47,000 simulated flights with no visible damage.

In the second test series the aim was to obtain at least three subsequent cracks in each panel. The number of flights from the initiation of the first crack until the second crack appeared, was on the average about 10,000 and further some 10,000 flights were required to reach the third crack. During such long periods the first crack would in most cases have propagated to complete failure of the panel. To prevent this and to avoid, as far as possible, any influence from the first crack on the initiation times of the following cracks, the first crack had to be stopped as early as possible, which was done by pressing steel balls, with a radius of 4 mm, from both sides against the sheet surface just in front of the crack tip. The procedure was successful for at least 10,000 flights, but it had to be repeated on several panels. The first stopping was made when the crack tip was 5 mm outside the edge of the strip, i.e. had a length of 15 mm from the centre of the rivet hole. A two-sided crack was thus allowed to reach a length of 30 mm. The same procedure was applied to the second crack, while the test was concluded upon the detection of the third crack. A tested panel with three cracks, two of them stopped, is shown in Fig 10. More detailed views are given in Fig 11, which were not taken from the same panel, the lower one having three stopped two-sided cracks.

Altogether eighteen panels were fatigue tested, one of which was cycled at a slightly lower stress level. Furthermore, one panel failed after about 50,000 flights, when the third crack had still not been initiated. Thus, 17 first and second cracks and 16 third cracks were recorded.

Due to improved method of load calibration the loadings applied were more accurate in the second series of fatigue tests than in the first one.

Crack initiation

The fatigue lives in number of flights T_{c1} until the detection of the first crack varied in the first test series between 13,000 and 55,000. Tabulating the logarithms of the lives the following mean value μ_{c1} and standard deviation σ_{c1} were obtained

$\mu_{c1} = 4.432$ corresponding $T_{c1} = 27{,}000$ flights

$\sigma_{c1} = 0.172$

The distribution curve for log T_c has been plotted in Fig 12 on Gaussian probability paper, using the 17 test results available.

The second test series with the same number of specimens gave the following logarithmic mean and standard deviation for the first crack initiation

$\mu_{c1} = 4.610$ corresponding $T_{c1} = 41{,}000$ flights

$\sigma_{c1} = 0.179$

Fig 13 shows the test points plotted in a log-normal diagram.

A comparison between the mean values shows an increase from 4.43 to 4.61 between the first and the second series, or from 27,000 to 41,000 flights. This extension of the fatigue life is probably mainly due to the fact that the stress distribution over the width of the later panels was more even, which means that the stresses at the inner rivet rows were somewhat lower than before. The bending in the sheet may also have decreased due to better stiffening of the panels. Furthermore, the sheet material, although of the same specification and with the same static strength, may have had better fatigue properties in the later delivery.

It is interesting to note that the standard deviation of log T_{c1} was almost unchanged, 0.179 compared to 0.172 in the earlier tests. The loads in the second test series were more accurately produced than in the first series, as stated above. The fact that the standard deviation has still not decreased implies that the scatter, represented by this quantity, is mainly due to the material properties, the design and manufacture of the test specimen and not so much to the accuracy of the testing machine. The larger variation of the stress over the width of the earlier panels implies that all four rivet rows are not equivalent. If this has any effect on the scatter, it would rather tend to give a larger standard deviation in the first test series.

The standard deviation obtained is rather large compared to results from other flight simulation tests on similar structures. This may be explained by the fact that the load spectrum was truncated at a rather low level in the upper end.

If the Weibull-distribution, Eq (19) is used insteal of the log-normal distribution, the two parameters T_o and α can easily be determined by plotting the test points on extreme value probability paper, fitting a straight line by eye. This has been done for the first and second test series in Figs 14 and 15 respectively. The following parameters were obtained

	First series	Second series
T_o	32,000	49,000
α	2.6	2.6

The three parameters of the complete Weibull-distribution, Eq (3), were determined by a least square method, searching the minimum on an electronic computer

	First series	Second series
T_u	11,100	230
T_o	20,400	49,600
α	1.52	2.53

The big difference in the lower limit T_u, which has also resulted in quite different shape-parameters α between the test series, is surprising and not easily explained from a physical point of view. This distribution gives the best fit to the test points, however.

The time to initiation of the second and the third cracks, T_{c2} and T_{c3}, has also been evaluated for the second test series, assuming a log-normal distribution, see Fig 13.

Second crack

$\mu_{c2} = 4.705$ corresponding $T_{c2} = 51{,}000$ flights

$\sigma_{c2} = 0.168$

Third crack

$\mu_{c3} = 4.776$ corresponding $T_{c3} = 60{,}000$ flights

$\sigma_{c3} = 0.130$

The 17 and 16 test points referred to the second and third cracks respectively, have also been plotted on extreme value probability paper in Fig 15, but no attempt was made to determine the Weibull-parameters.

If a structural component includes a number m of identical and independent fatigue sensitive members or spots, say rivet holes with load transfer, the probability P_{c1} of initiation of the first crack can be obtained from the "parent" distribution P_c by using the Bernoulli theorem, see HELLER-HELLER[57]

$$P_{c1} = 1 - (1 - P_c)^m \tag{23}$$

Also the probabilities P_{c2} and P_{c3} of the second and third crack initiations can similarly be derived as functions of the probability P_c

$$P_{c2} = P_{c1} - mP_c(1 - P_c)^{m-1} \tag{24}$$

$$P_{c3} = P_{c2} - \frac{m(m-1)}{\underline{|2}} P_c^{\,2} (1 - P_c)^{m-2} \tag{25}$$

The parent probability P_c may be solved explicitly from Eq (23). This is not possible in Eqs (24) and (25), but P_c can be evaluated numerically as functions of P_{c2} and P_{c3} respectively. The separate curves for the initiation of the first, second and third cracks of Fig 13 have thus been converted, assuming m = 32, into one single curve in a log-normal diagram in Fig 16 giving the probability of crack initiation at a single stress concentration.

The distribution of the parent population in Fig 16 contains 17+17+16=50 test points from the first, second and third cracks, which are distinguished by separate symbols. If the assumption holds true that the fatigue sensitive spots are equivalent and independent, all the points should fall along the same curve, which would be a straight line in case the parent distribution is log-normal. A close study of the three sets of test points reveals that they form three separate curves, which are all nearly straight lines, the lowest representing the first crack and the highest the third crack. The explanation seems to be that the time to initiation of the later cracks was influenced to same extent by the initiation and propagation of earlier cracks. The forming of the first crack obviously causes same stress redistribution in the vicinity of the crack. Although it has not been observed that the second and third cracks often occur in areas where the stresses have been increased due to the first crack, the diagram tends to indicate such an effect. The second crack has probably been initiated a few thousand flights too early, and it has also caused a further reduction of the life until the third crack.

If the distribution function of the number of flights to the first crack is log-normal as assumed in Figs 12 and 13 the parent distribution, obtained from Eq (23), will not be log-normal. This is demonstrated in Fig 12, where parent distributions have been computed taking either all stress concentrations, m = 32, into account, or only those of the two inner rivet rows, m = 16. The distribution of the second crack initiation has also been evaluated for m = 32. It is rather close to a straight line. Employing instead the Weibull-distribution for the first crack, as in Figs 14 and 15, it is easily seen from Eq (23) that also the parent distribution will become a straight line in the log-extreme value diagram, parallel to the first crack distribution. The distribution curve of the second crack is not a straight line, however, and is also considerably steeper.

Although it may seem attractive to obtain the parent distribution by drawing a straight line parallel to the distribution of the first crack, this mathematical relation does not in itself speak in favour of the Weibull-distribution. It is not possible to draw any significant conclusions regarding a comparison between the two two-parameter distributions used, on the basis of the test results obtained in the first and second test series presented. The slightly better fit obtained with the log-normal distribution would not necessarily be repeated in a new similar investigation. Due to this uncertainty with respect to distribution function it must also be concluded that the rather numerous fatigue lives to crack initiation obtained, do not allow a safe extrapolation of the probability of the initiation of the first crack further down than $P_{c1} \approx 1$ per cent.

Crack detection

The times until crack initiation presented and discussed above are strictly the times recorded when a crack was detected. The visible length of the cracks varied between 0.5 and 20 mm when they were found. Since the lengths of all detected cracks have been recorded and also the crack propagation has been determined, it would be possible to correct the crack detection times to strict crack initiation times, defining the initiation of a macro-crack as the moment when it reaches a fixed length. A study has indicated that such a correction procedure would not bring any noticeable change of the distribution curves for crack initiation. Considering the whole service life until crack initiation the length of a detectable crack is consequently rather unimportant. This is not the case when one is trying to evaluate the safety of a structure, basing the safety on crack detection at rather frequent inspections.

Altogether 49 cracks were recorded in the first series of fatigue tests and 106 in the second series. Since the automatic crack wire system had been improved from the first to the second test series, the recordings of crack lengths detected were treated separately, see Figs 17 and 18. One could also have distinguished between the first cracks which were always detected by crack wire, and subsequent cracks, where visual inspection had to be relied on if the wire could not be repaired. However, such a separation was not undertaken. As explained by small sketches in Figs 17 and 18 the crack length visible on one side of the countersunk rivet head is denoted a_s. This quantity was measured for all cracks found on the panels. After arranging the recorded values in order from the lowest to the highest, the probability of detection was computed and plotted versus the respective crack lengths in a log-normal diagram as shown in Figs 17 and 18. It is possible to fit straight lines to the test points, especially for the first series.

Since one is mainly interested in the total length L of a crack from tip to tip, it was assumed that each crack was symmetrical with respect to the centre line through the rivet. The diameter of the countersunk rivet head was 10 mm, and the total length of this thought crack could thus be determined as $L = 2\ a_s + 10$ mm. The distributions of L have also been plotted in Fig 17 and 18. A smoothed distribution curve for crack length L was obtained from Fig 18 for the second test series. It has been drawn in Fig 19 with a linear probability scale versus L in a logarithmic scale.

Crack propagation and residual strength

A considerable scatter was found in crack lengths when these were plotted versus the number of flights after crack detection[11]. This scatter disappeared almost completely for short crack lengths up to 25 mm when the propagation curves were brought to start at a common point with a fixed crack length. It was thus possible to obtain a representative mean curve for the test panels. Fig 20 shows the growth of crack length L from tip to tip with time t_d from the crack initiation, which is assumed to occur for L = 10 mm, the minimum crack length that could possibly be detected by crack wire or visual inspection.

Introducing numerical values of the crack propagation time t_d for a number of crack lengths L from Fig 20 into the crack detection diagram of Fig 19, the probability of crack detection F_d may be plotted versus t_d, as shown in Fig 21. The scatter present in crack propagation after 3000-4000 flights may probably be neglected, since the number of cracks detected at that time after crack initiation are approaching 90 per cent and the distribution curve is rather flat.

By loading 17 stiffened panels with cracks of lengths L ranging from 10.5 to 91 mm to ultimate static failure, the residual strength U normalized with respect to its original value U_o was determined as a function of L, Fig 22. The scatter of the residual strength is obviously quite small. Introducing values from the crack propagation curve, Fig 20, into the residual strength diagram of Fig 22, the normalized residual static margin r, according to Eq (8), may be plotted versus the crack propagation time t_d, see Fig 23. A mean curve has been fitted to the test points. For $t_d > 6000$ flights it has been dotted. Using the crack propagation curve of Fig 20 with an asymptote at about 6500 flights, would give a vertical decrease in strength at that time. But the crack propagation curve was determined on panels without continuous stiffeners and is thus unrealistic for long cracks approaching the distance between the stiffeners. In a panel with the configuration used for the ultimate static tests, the stiffeners would have acted as crack stoppers giving a constant residual strength during a large number of flights, as indicated in Fig 23. Also two linear strength decrease curves have been drawn which descend to zero in 10,000 and 15,000 flights respectively. In order to achieve low probabilities of failure one has to carry out regular inspections at intervals which are considerably shorter than 6000 flights. The shape of the residual static margin curve is consequently of less importance for $t_d > 6000$ flights in a reliability analysis where expected fatigue damage is taken into account. Crack stoppers, on the other hand, are useful to safeguard against "inconceivable" events.

7. NUMERICAL CALCULATIONS OF PROBABILITY OF FAILURE

In earlier publications[7,11] a more simplified approach was used at FFA in order to produce a diagram giving the probability of fatigue failure versus the service life in hours of flight or number of flights, assuming periodic inspection with a number of different constant intervals. The parameters introduced in the numerical calculations were the following:

Load spectrum, heavy gusts, Eq (7)

$H_o = 0.2 \quad h = 20$

Crack initiation, log-normal distribution, Eq (2)

$\mu_c = 4.699 \qquad \sigma_c = 0.2$

Normalized static margin, deterministic

$r(t_d) = 1 - t_d/10{,}000$

These values were obtained from the literature and experience of own testing. The numerical calculations of the present report assume the same load spectrum for heavy gust loads which seems to be quite realistic for a jet transport aircraft on routes with a flight time of about 1 h. The information required concerning crack initiation, strength reduction and crack detection have been taken directly from the results of the panels tests carried out at FFA. If the sheet panels which were subjected to fatigue testing and ultimate static tests, may be considered representative for the most fatigue-sensitive parts of a wing structure, the results of the calculations should give probabilities of failure which are at least of the correct order of magnitude. The calculations have not been performed, however, with the aim of establishing direct design information, but rather in order to give an idea of the difficulties involved in a more sophisticated analysis.

From the second test series the following crack initiation parameters were taken:

$\mu_c = 4.60 \qquad \sigma_c = 0.18$

Furthermore, the probability of crack detection F_d versus number of flights t_d after crack initiation presented in Fig 21 was introduced in 12 discrete points between 0 and 5500 flights. The test results from the first series of the residual static margin, plotted in Fig 23 as a function of t_d, have been approximated by a polynomial

$$r(t_d) = C_o + C_1(t_d\ 1000) + C_2(t_d/1000)^2 \qquad (26)$$

where the constants were determined by a least square method

$C_o = 0.9053 \qquad C_1 = -0.3766 \cdot 10^{-2} \qquad C_2 = -0.5482 \cdot 10^{-2}$

This parabola intersects the abscissa at about 12500 flights. Until $t_d \approx 6000$ flights it follows the full-drawn curve in Fig 23, while for higher values of t_d it falls between the vertical decrease of a sheet without stiffeners and the horizontal line indicating crack arrest by longitudinal stiffeners.

It was assumed that the residual static margin is normally distributed with a coefficient of variation of 0.05, implying a somewhat lower coefficient of variation for the ultimate residual strength.

The conditional probability of failure $G(t_d)$, provided that a crack has been initiated in the panel, was computed introducing the residual strength reduction curve Eq (26), with assumed Gaussian distribution function, and the load spectrum of extreme turbulence. This probability has been plotted in Fig 24 versus the number of flights t_d after crack initiation. It should be noted that for values $t_d > 6000$ flights the G-function is only of theoretical interest, since it is no longer based on test data.

By numerical integration of Eqs (12) - (16) in generalized form the probability of fatigue failure during a service life of T flights was finally obtained for constant inspection intervals of 1000, 2000, ..., 8000 flights, as shown in Fig 25. Inspection intervals longer than 6000 flights should not be considered in a structure with a total crack propagation time of about 6500 flights. If the service life is limited to 20,000 flights i.e. half the logarithmic mean life, corresponding to $P_c \approx 0.05$, the probability of failure is still quite low $P \approx 10^{-5}$ for an inspection interval as long as 5000-6000 flights. An inspection interval of 3000 flights which is about half the crack propagation time, seems to give a low P-value even if the service life is extended to the mean life. It should be remembered, however, that the distribution curve of detected crack length has been obtained from laboratory observations. The probability would probably be considerably higher especially for comparatively long inspection intervals, if a distribution representative of scheduled service inspection were introduced instead.

The numerical calculations will be continued in order to study the influence of different parameters

8. CONCLUSIONS

A review has been made of the probabilistic assessment of the safety of a damage tolerant wing panel structure subjected to scheduled service inspections. It was concluded that the fatigue life to crack initiation, including scatter, and the crack propagation life cannot generally be determined by analysis or by relying on past experience condensed in empirical formulae. A new wing design will require extensive flight simulation testing, where it is not an easy task to decide on the truncation of load spectra and on variation of loadings between flights. It was also found that very few empirical investigations on detectable crack lengths are available in literature. There is an urgent need for data concerning scheduled service inspection.

Results from two series of fatigue tests on similar wing panels have been presented, which include the information needed to make an analysis of the probability of failure. This has been used in numerical calculations resulting in Fig 25, which gives the probability of fatigue failure during the service life of a transport aircraft, assuming periodic inspections with constant intervals of lengths from 1000 to 8000 flights. If the length of the inspection interval is restricted to half the crack propagation time, the diagram suggests that the probability of failure will never exceed 10^{-5}. The crack propagation time is then defined as the number of flights from the initiation of a macro-crack until it has reduced the strength of the panel to a value approximately amounting to the limit load. This result is strongly dependent on the crack-length-detection diagram established from the laboratory inspections. Scheduled service inspections with lower probability of detecting a given crack length would result in higher failure risks. It is consequently of great importance to define in inspection criteria what should be expected from various NDI techniques.

Since the analysis was based on laboratory fatigue tests on panel specimens manufactured in a laboratory model workshop neither environmental effects in service, such as corrosion, nor defects met in production manufacture, nor engineering approximations and mistakes, have been included in the results presented in Fig 25. In an actual design case one will have to introduce extra margins of safety by safety factors on the service life, by increasing the coefficients of variation with regard to unknown variabilities, or by requiring a probability of failure which is one or several factors of ten lower than would normally be considered satisfactory. Even with the best statistical information one could hope to collect, the safety problem will never be solved by exact mathematical analysis. Subjective elements have to be mixed into the probabilistic methods as is practised in the Bayesian decision theory.

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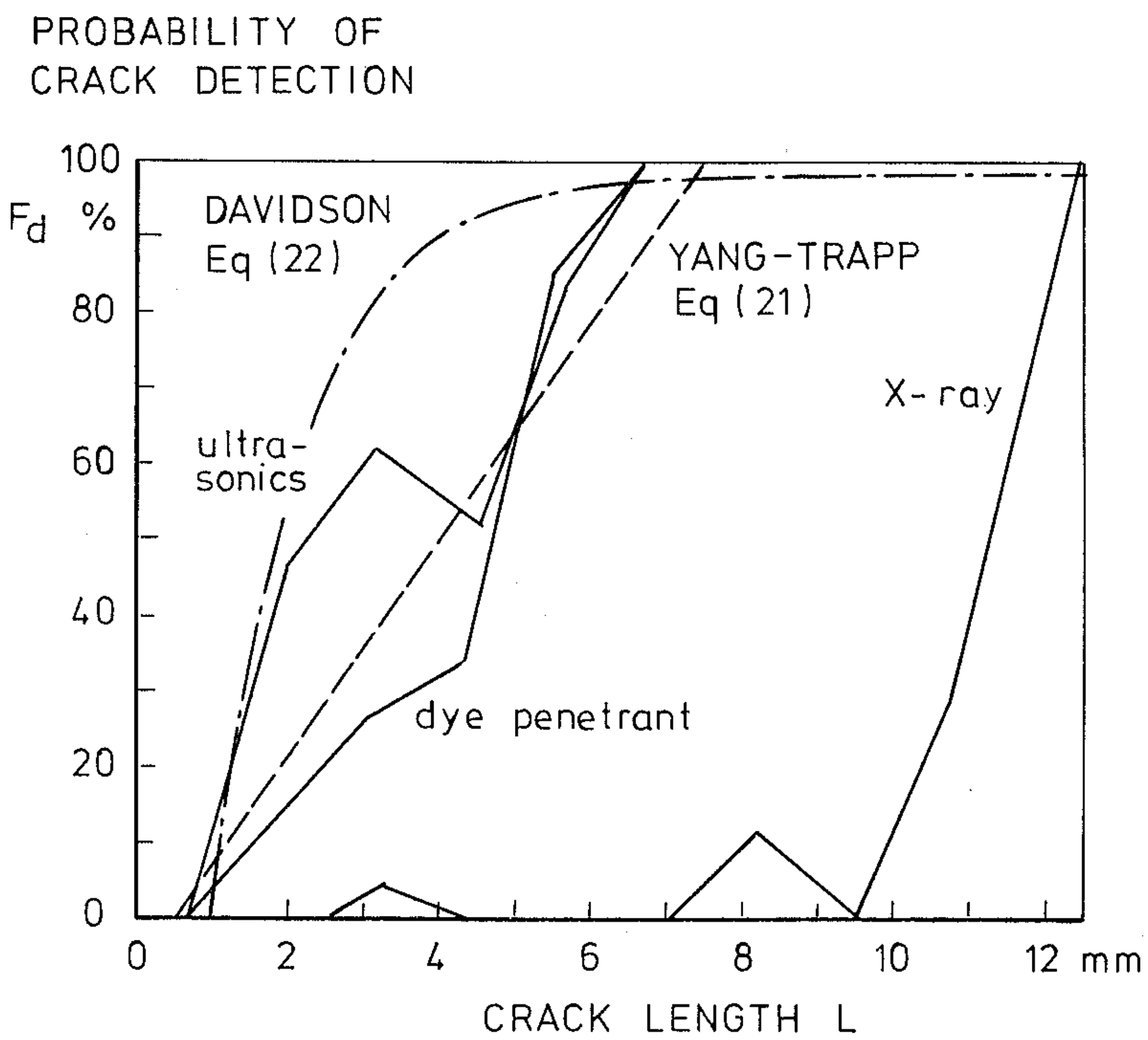

Fig. 1 Probability of crack detection for 7075 aluminium specimen determined in laboratory fatigue testing

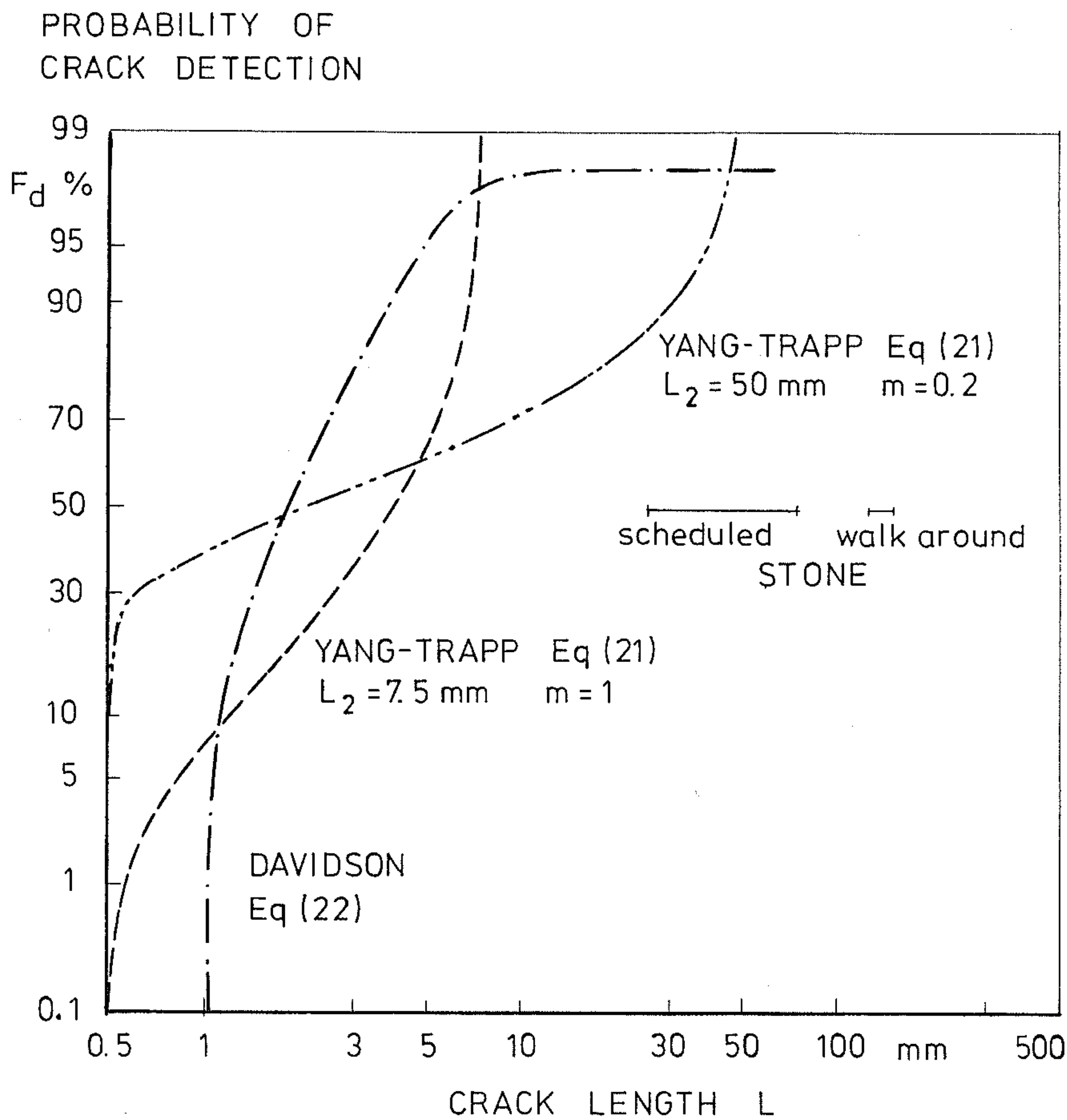

Fig. 2 Proposed distribution functions for crack detection lengths plotted in a logarithmic - Gaussian diagram

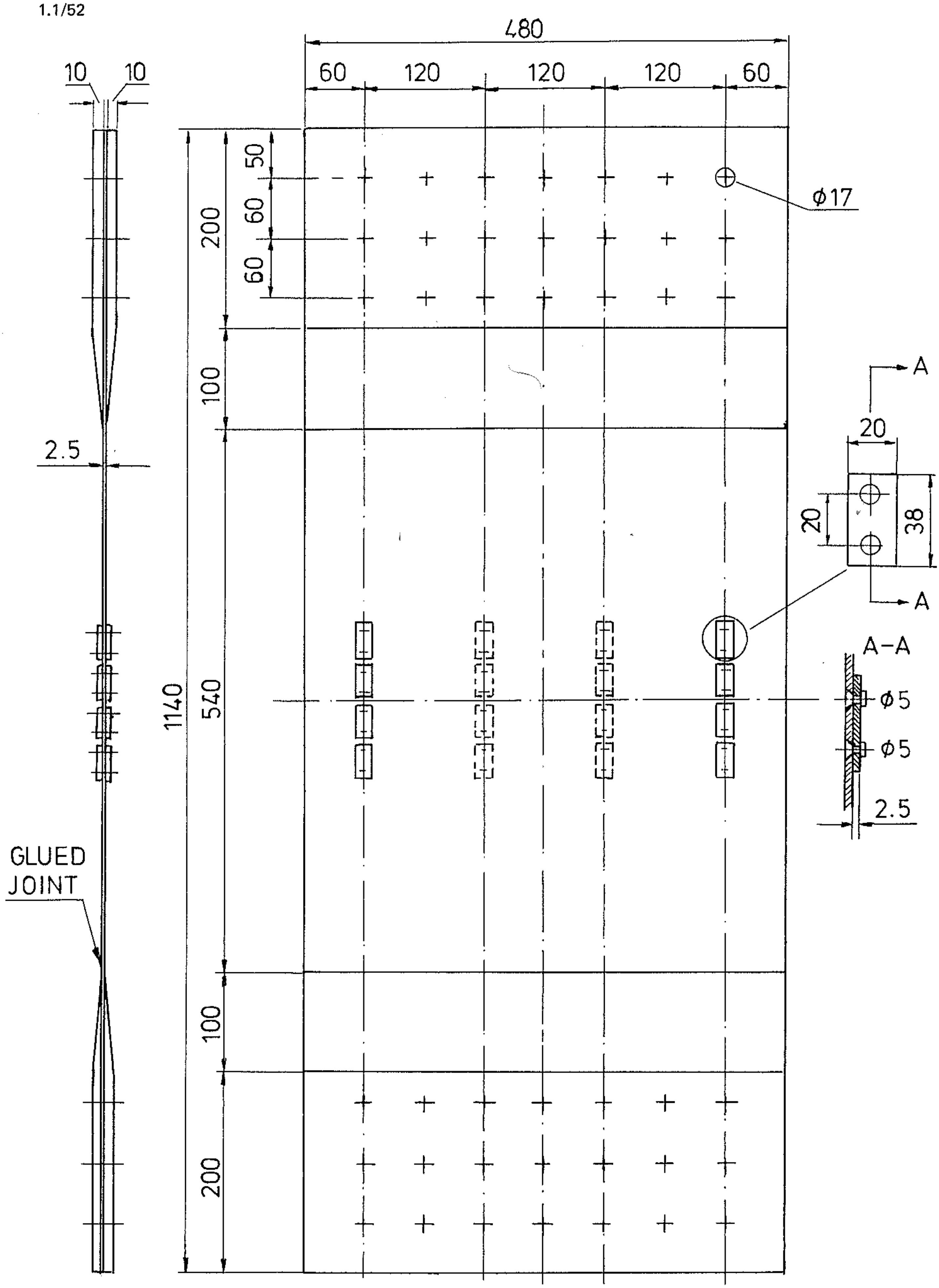

Fig. 3 Panel for fatigue testing, first test series.
Dimensions in mm

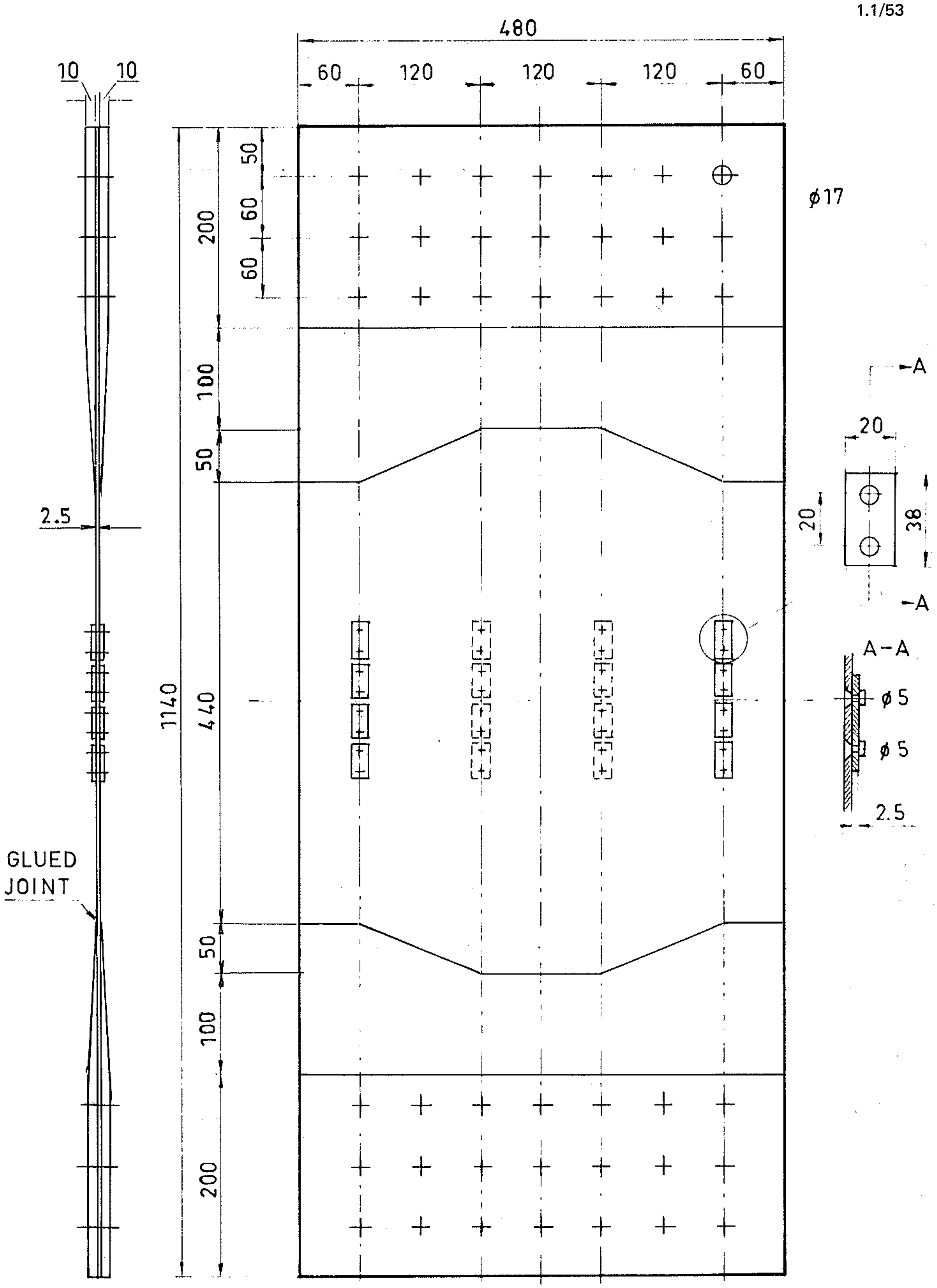

Fig. 4 Panel for fatigue testing, second test series.
Dimensions in mm

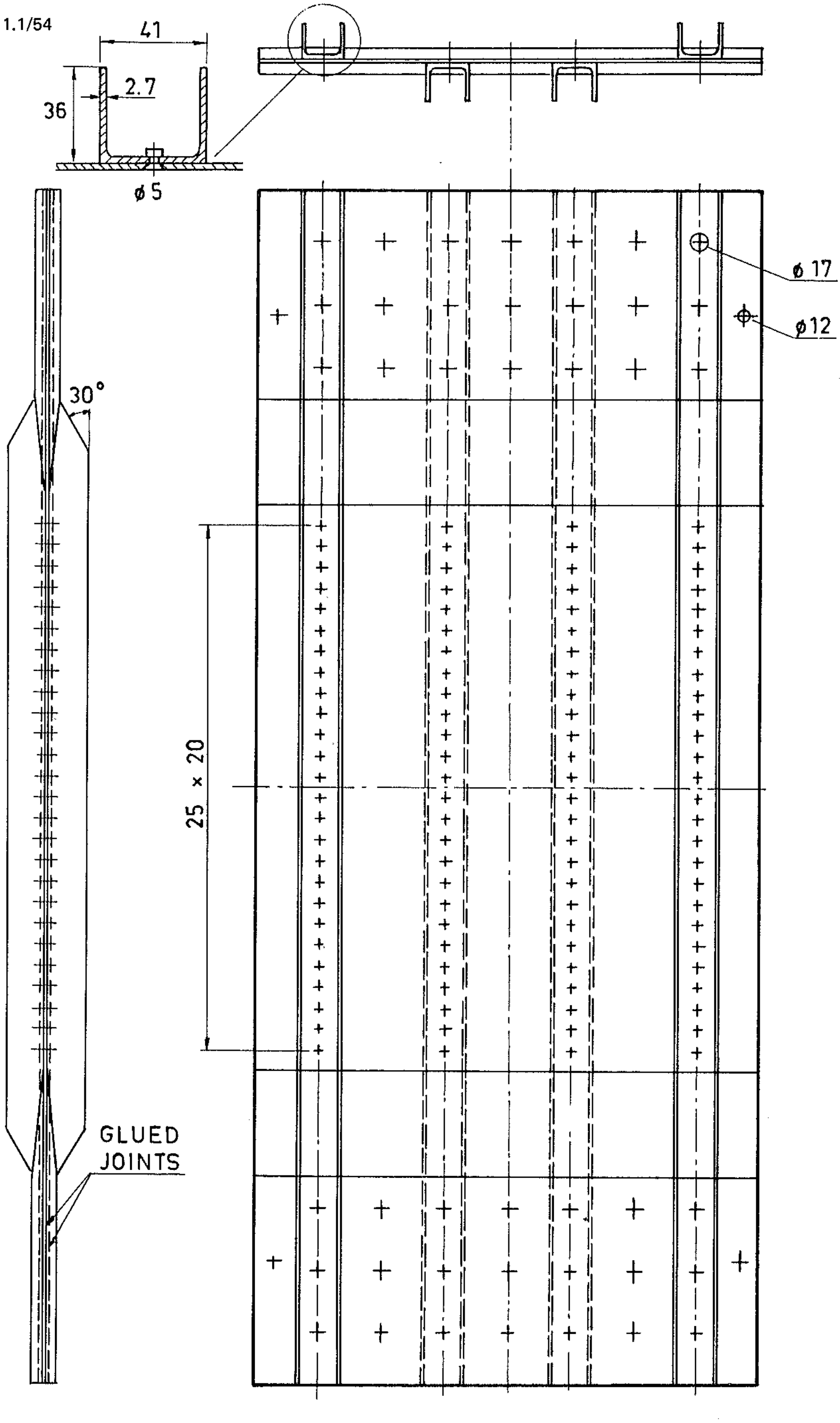

Fig. 5 Panel for ultimate static tests, dimensions in mm.

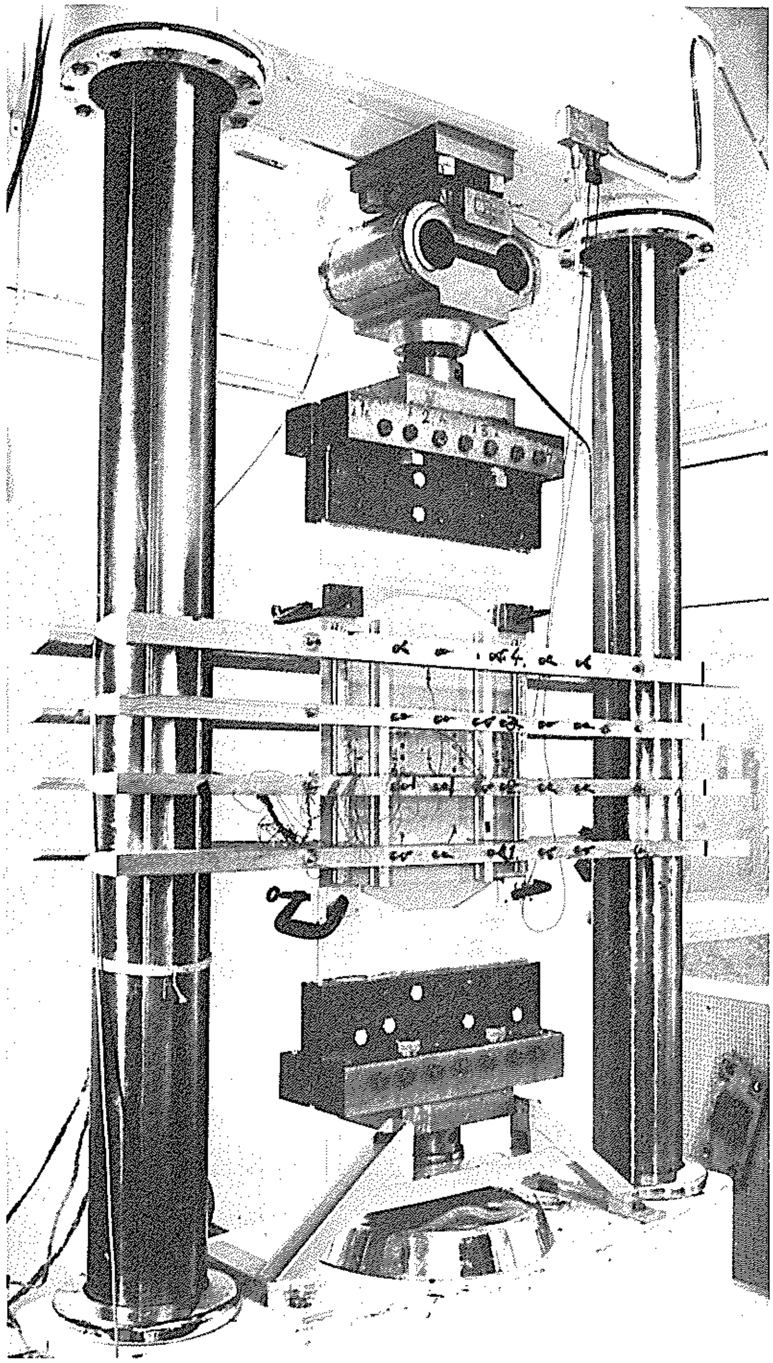

Fig. 6 Fatigue testing machine with test panel stiffened to prevent buckling

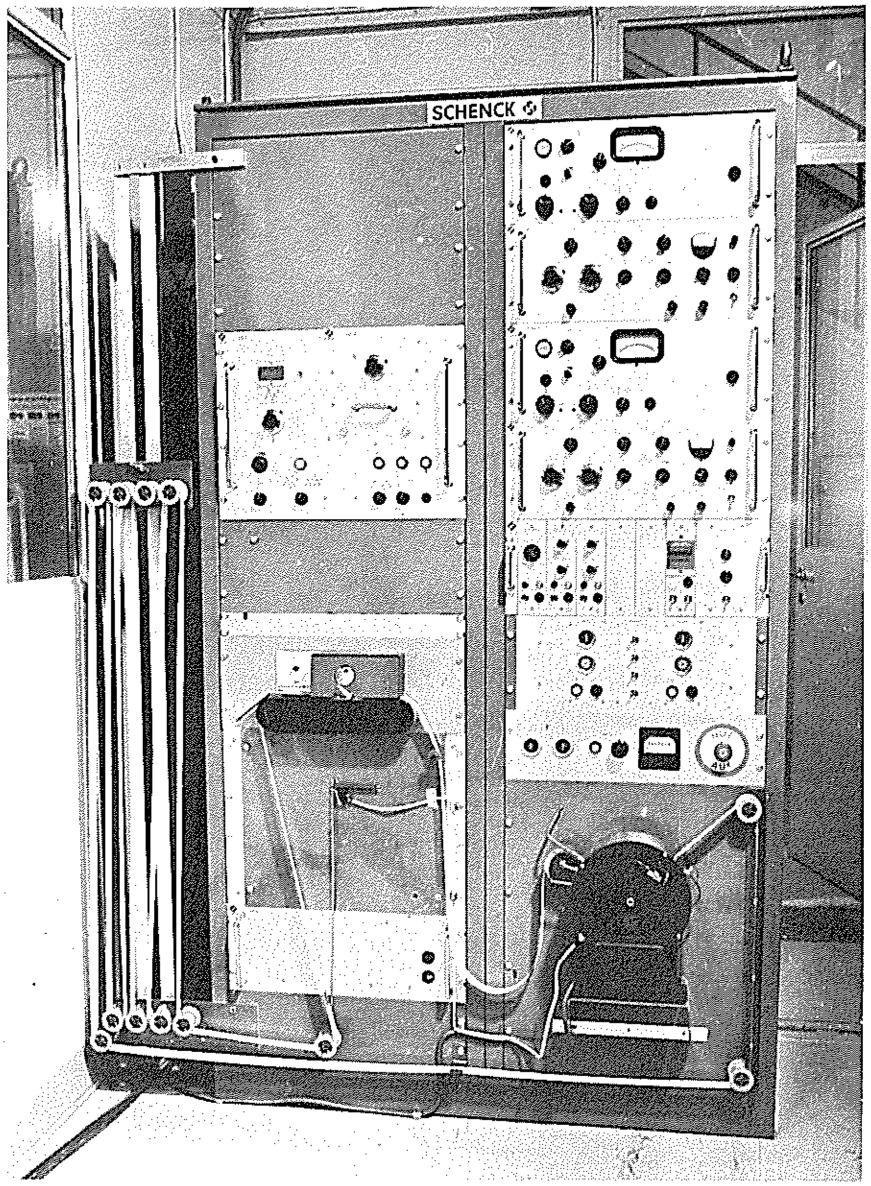

Fig. 7 Punched-tape-operated programme unit

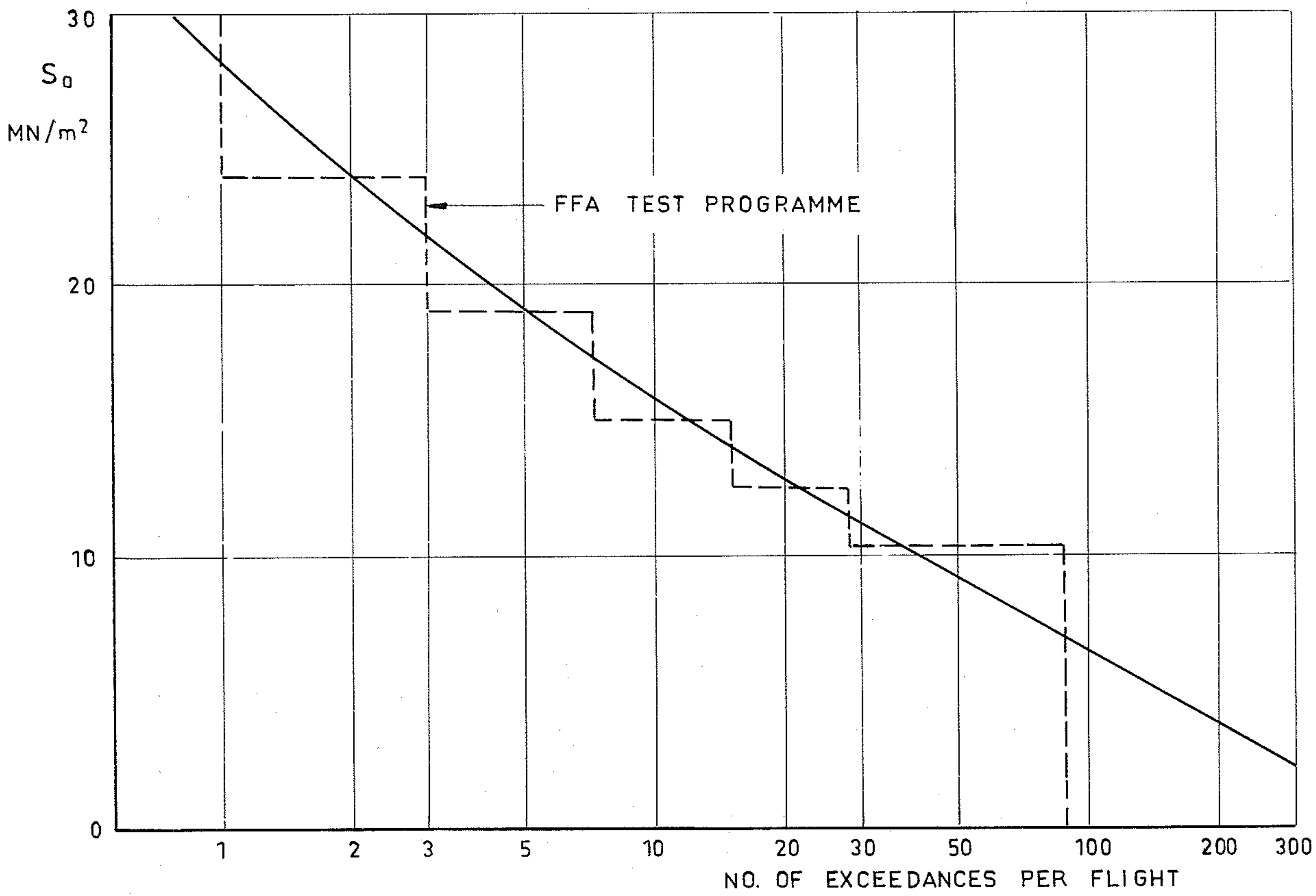

Fig. 8 Stress amplitude exceedance distribution. Typical 1-h flight with civil transport airplane.

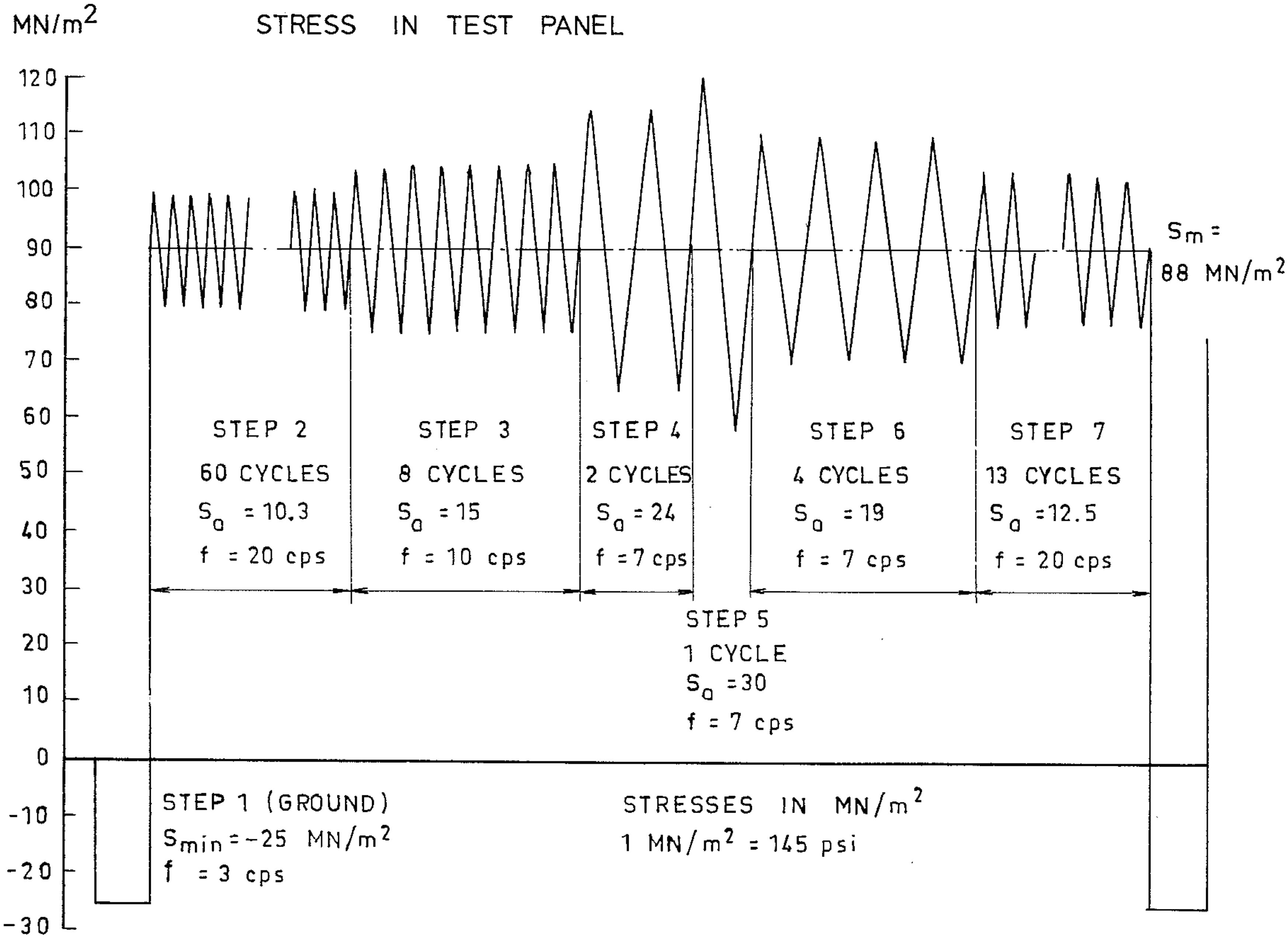

Fig. 9 Load sequence for simulation of 1-h flight in fatigue test

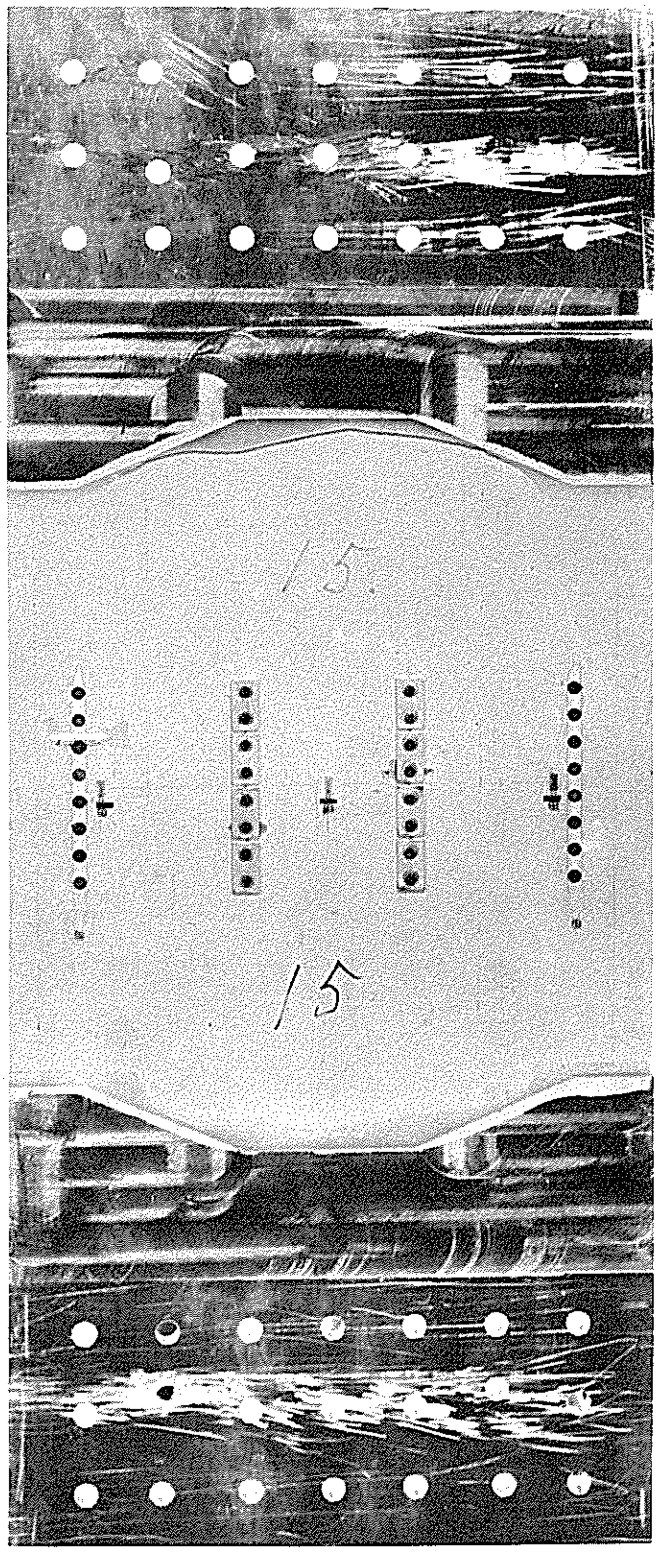

Fig. 10 View of tested panel with three cracks, two of which have been stopped

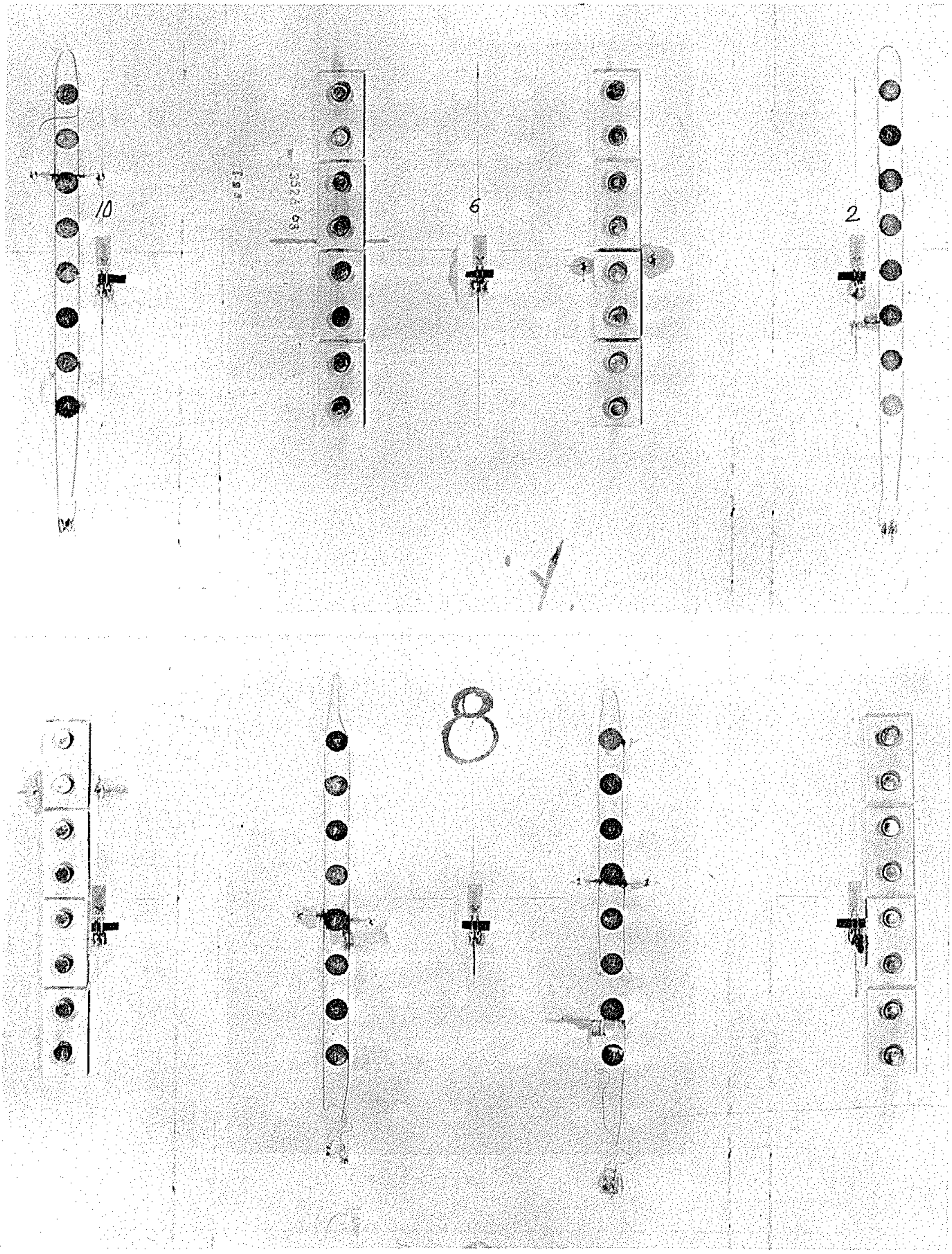

Fig. 11 Detail views of different sides of two tested panels with cracks stopped by ball pressure

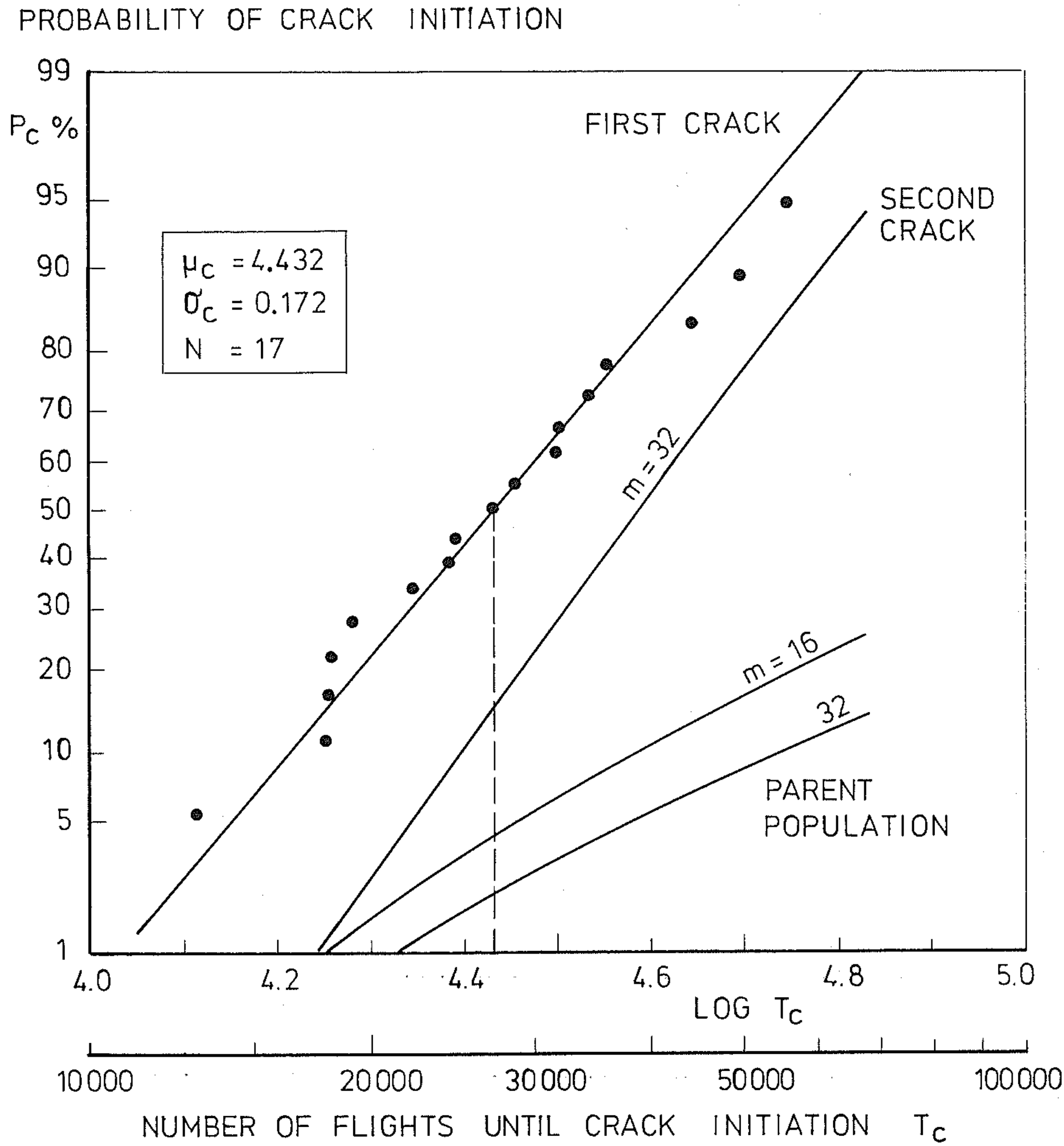

Fig. 12 Distribution curve on Gaussian probability paper for number of flights to initiation of first crack, obtained from 17 panel fatigue tests. Distribution curves for parent populations have been computed assuming number of identical members m = 16 and 32. Distribution of second crack initiation determined from parent distribution m = 32

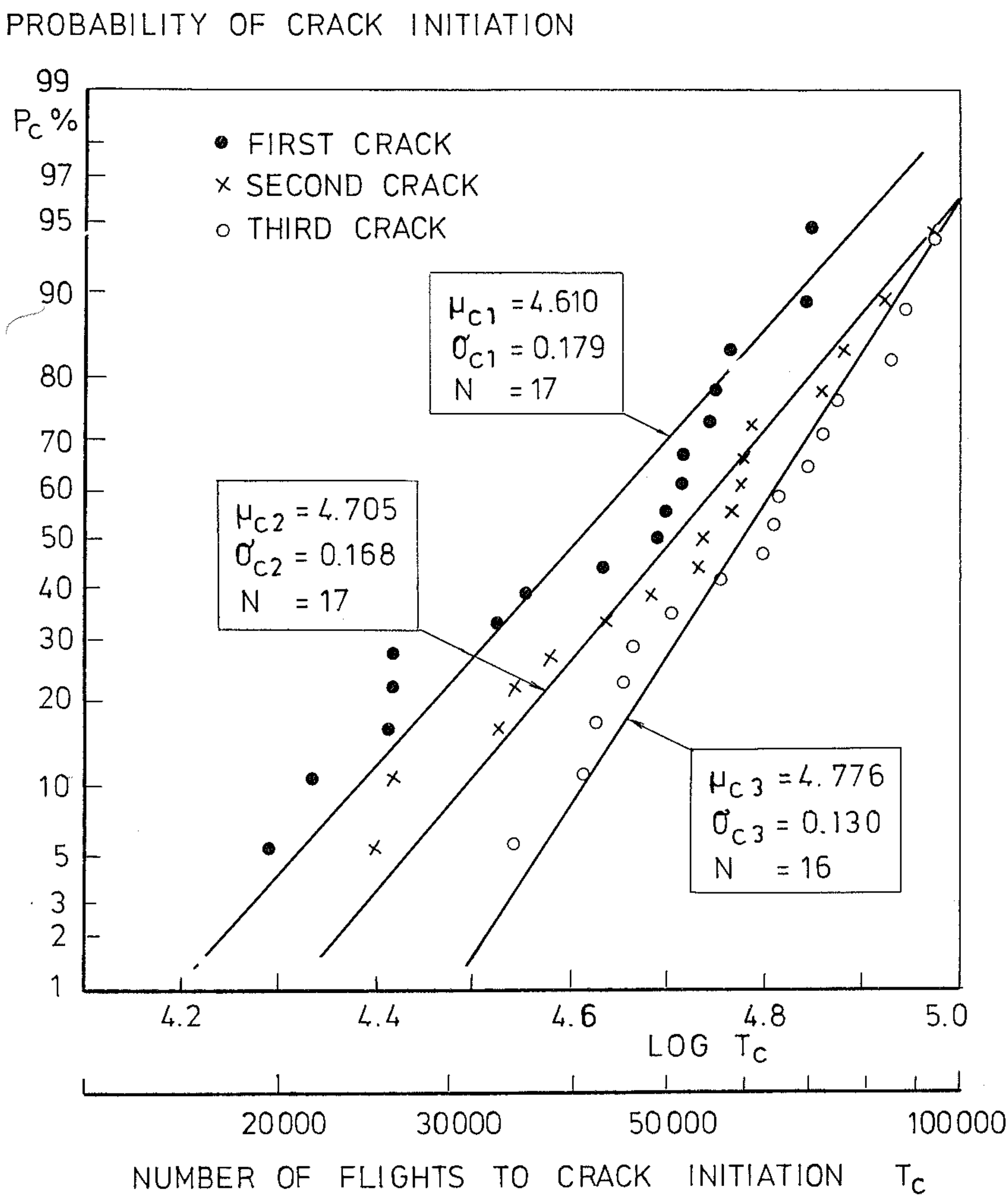

Fig. 13 Test results plotted on Gaussian probability paper for first, second and third crack initiation

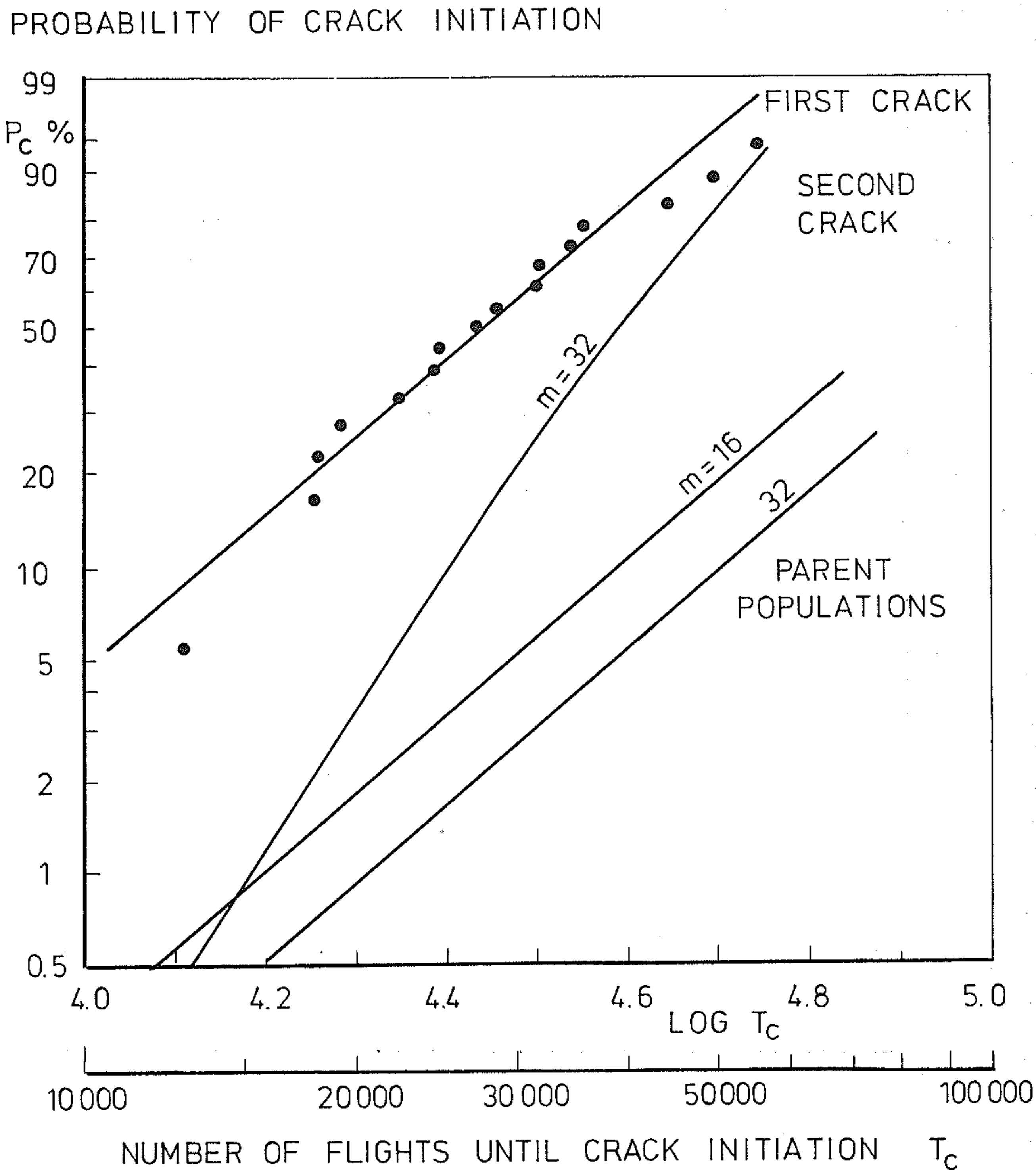

Fig. 14 Distribution curve for number of flights to initiation of first crack, obtained by plotting test results on extreme value probability paper. Distribution curves for parent populations, assuming m = 16 and 32, and distribution of second crack initiation with m = 32, have been computed

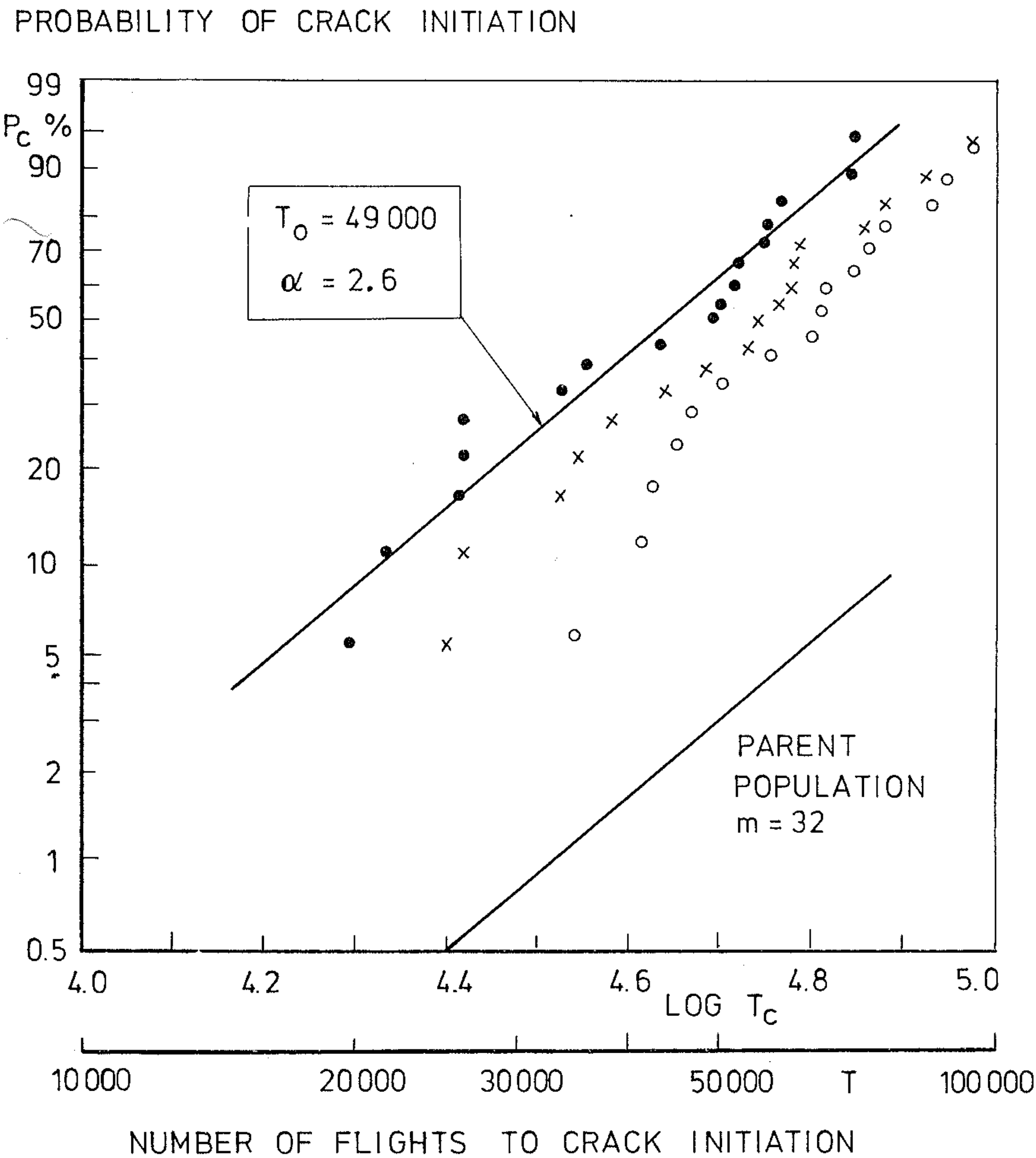

Fig. 15 Test results plotted on extreme value probability paper for first, second and third crack. Distribution curve for parent population obtained assuming 32 stress concentrations

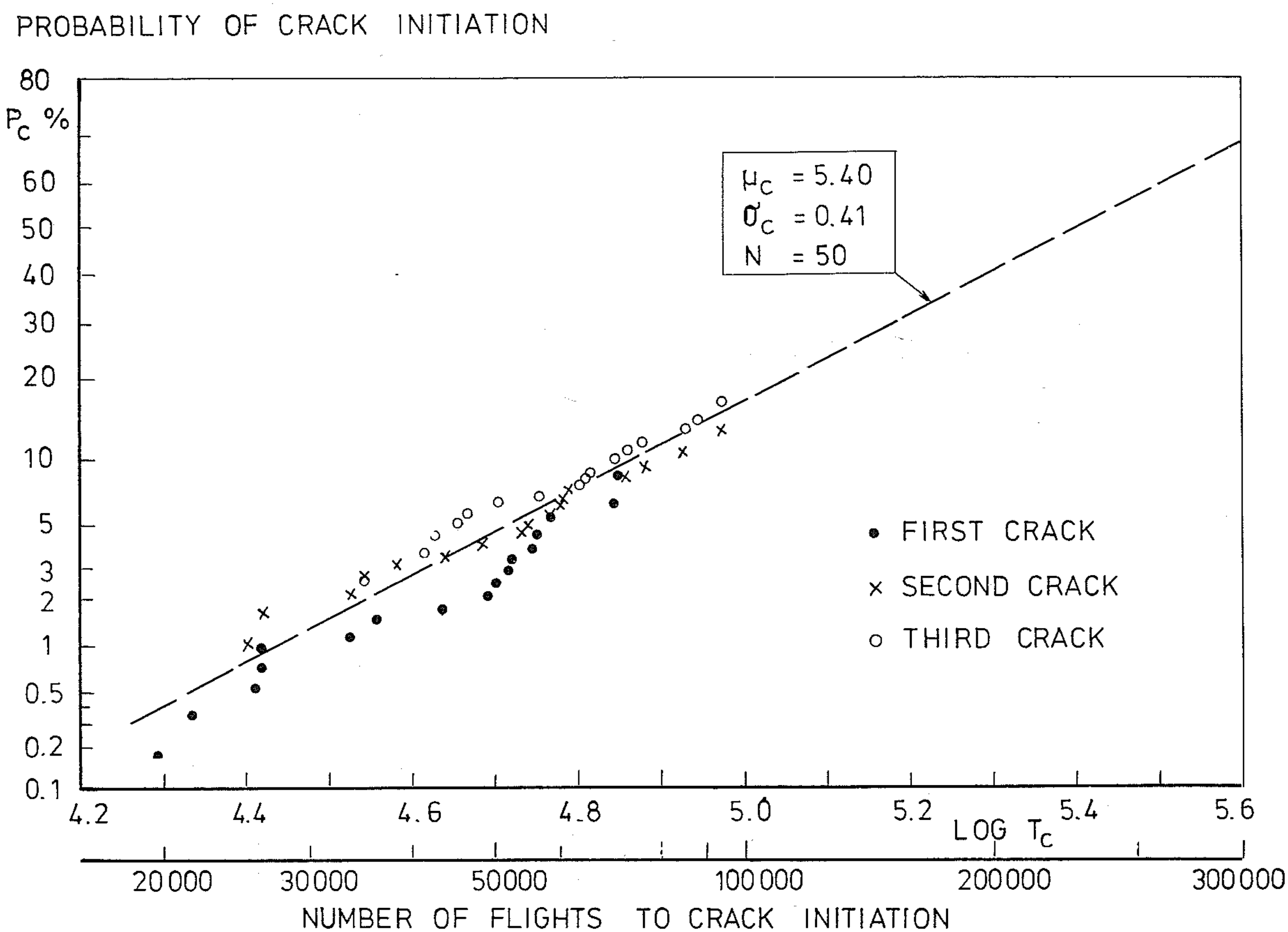

Fig. 16 Distribution curve for parent population computed from observations for first, second and third crack assuming 32 equivalent and independent stress concentrations. Gaussian probability paper used

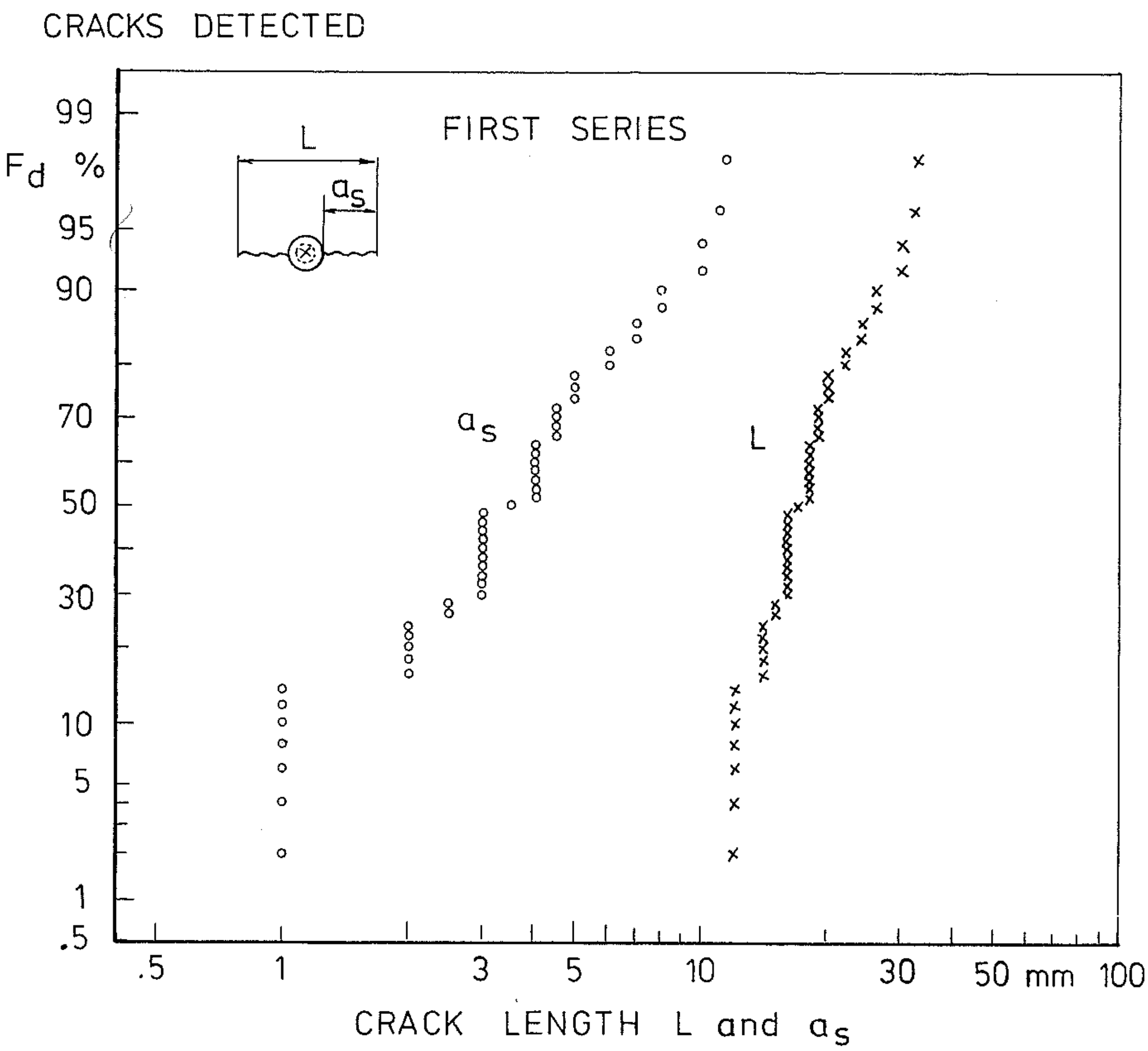

Fig. 17 Distribution curves for detected crack lengths obtained from first test series by plotting measured values on Gaussian probability paper

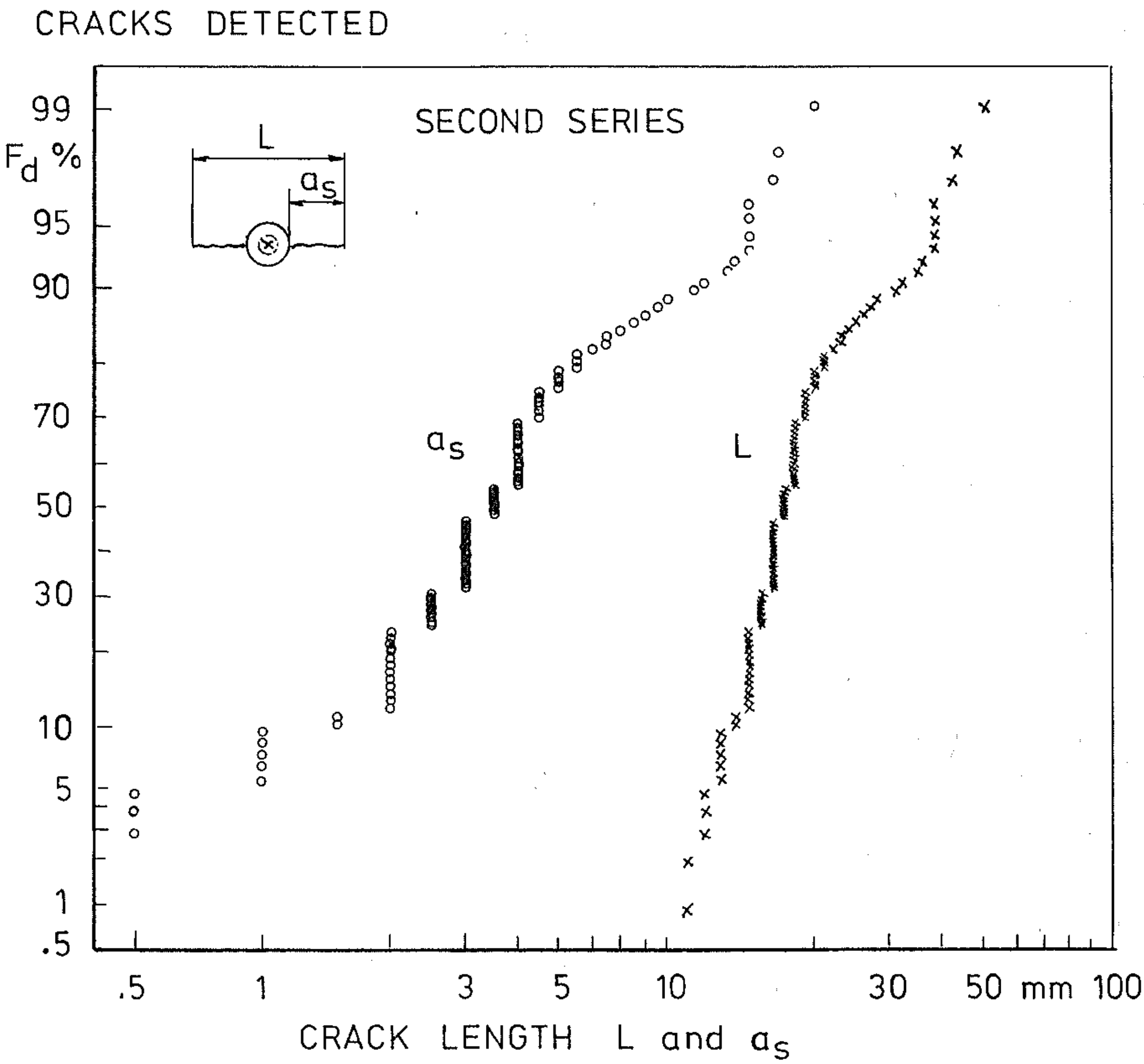

Fig. 18 Distribution curves for detected crack lengths obtained from second test series by plotting measured values on Gaussian probability paper

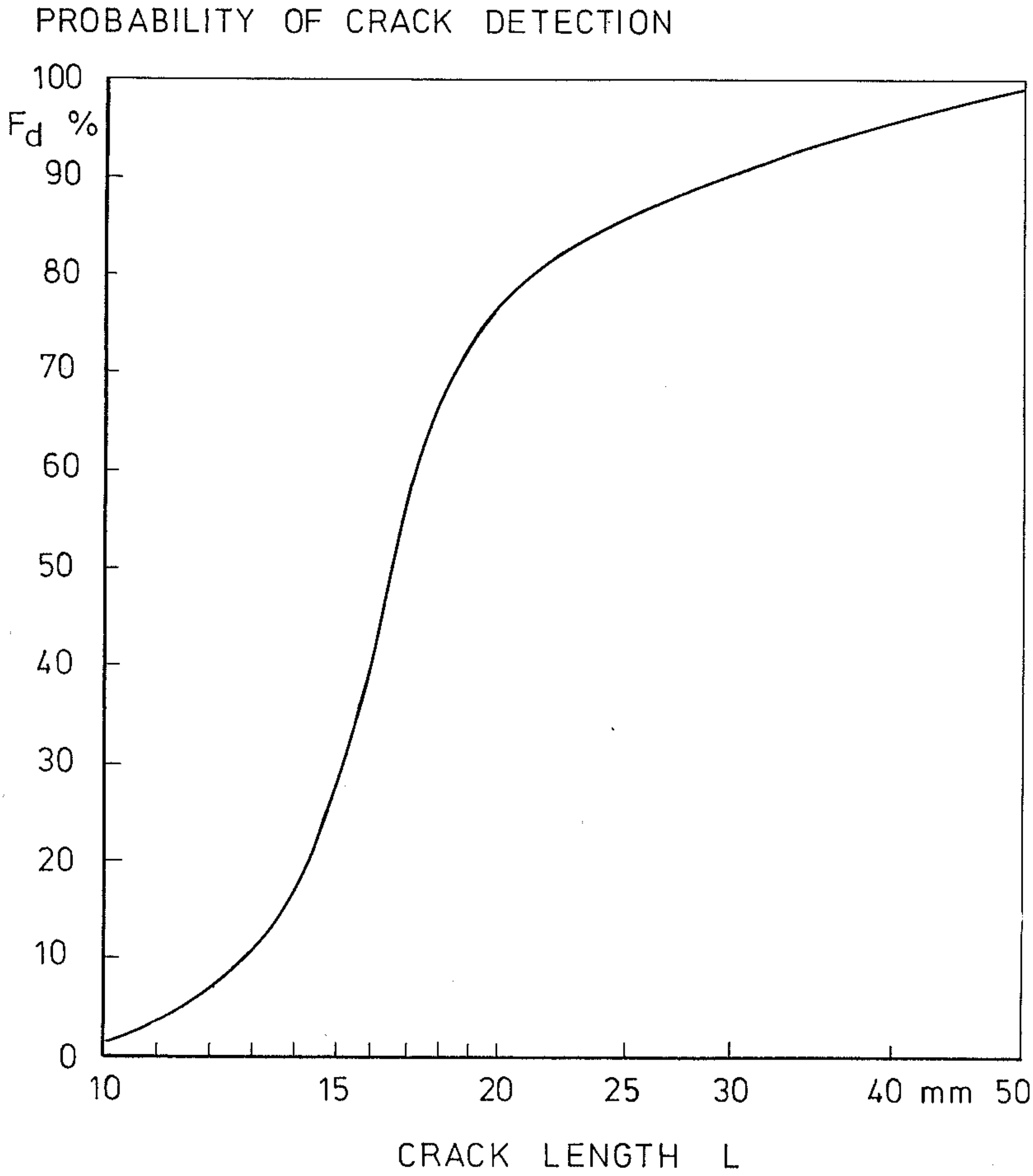

Fig. 19 Probability F_d of detection of crack length L

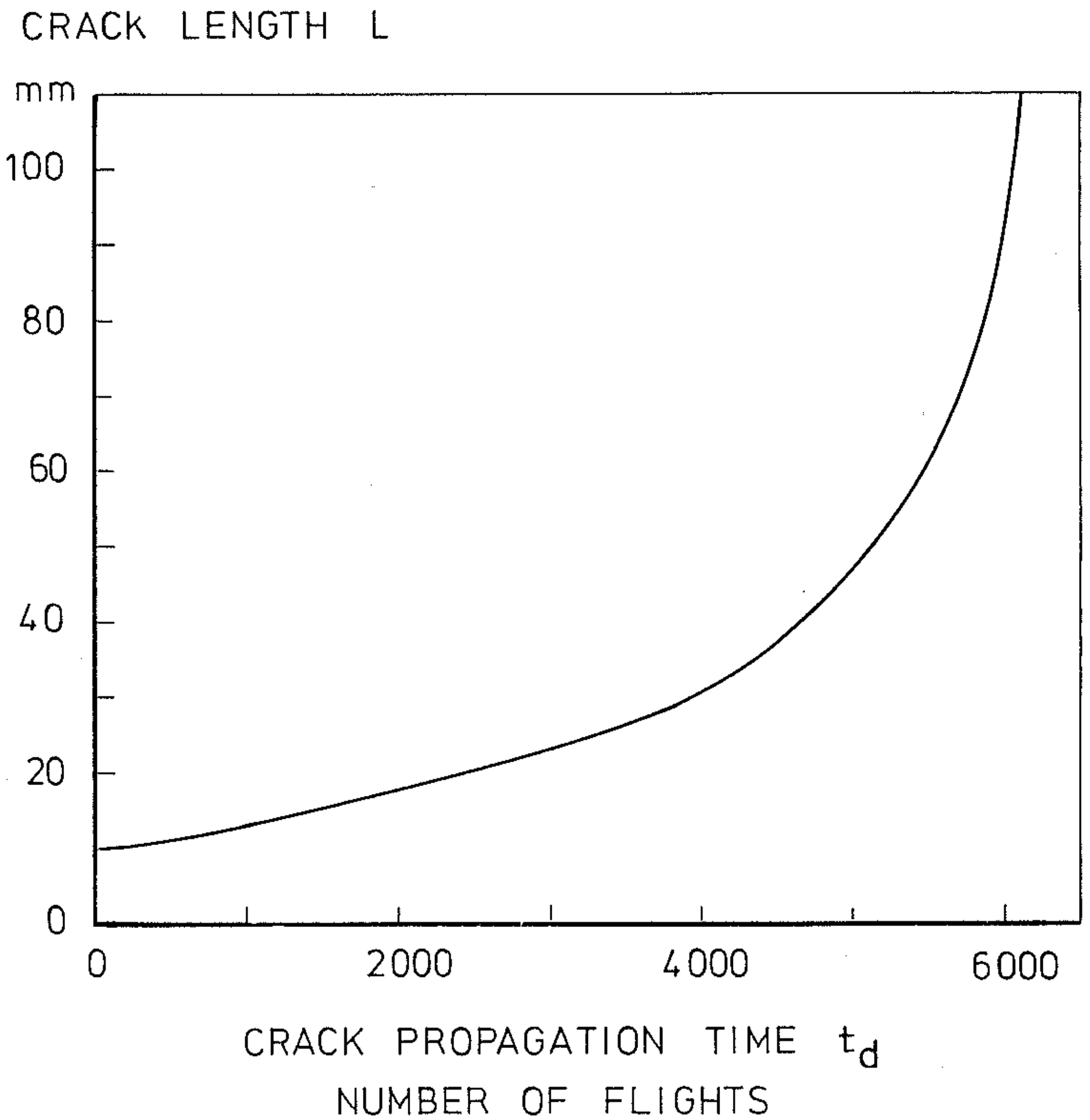

Fig. 20 Mean crack propagation curve determined from test panels

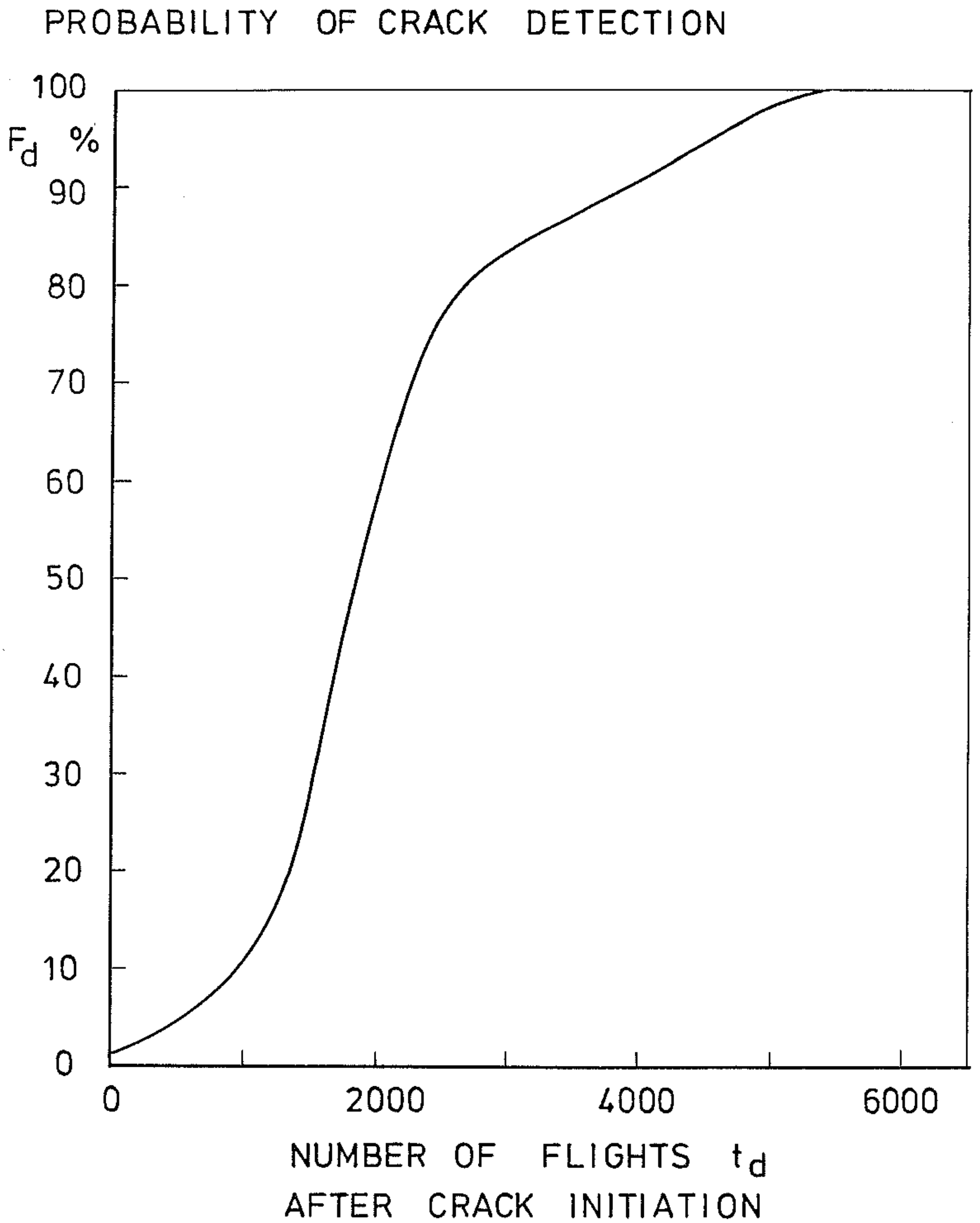

Fig. 21 Probability F_d (t_d) of detecting crack at time t_d after initiation

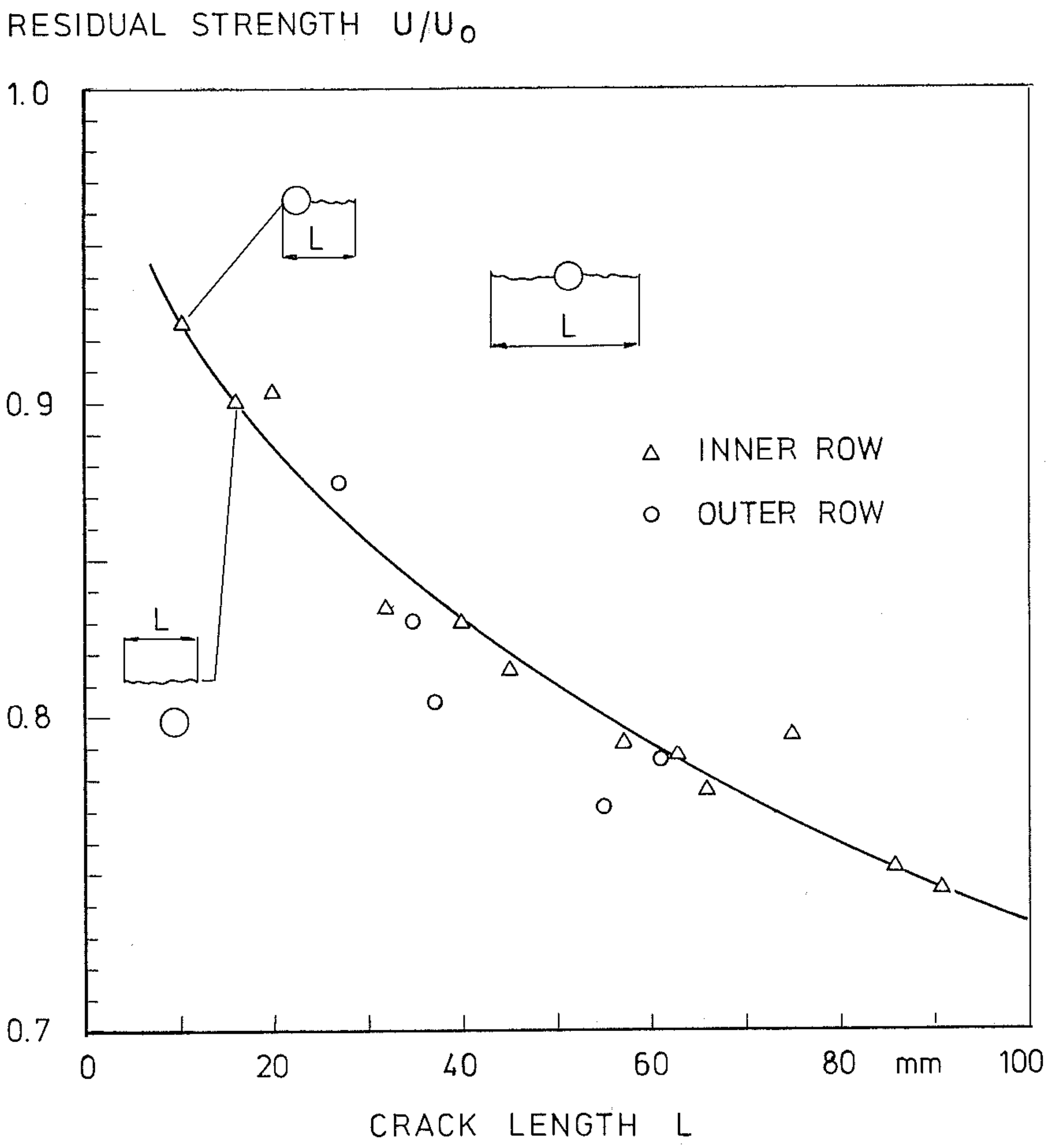

Fig. 22 Residual strength of damaged panel plotted versus crack length

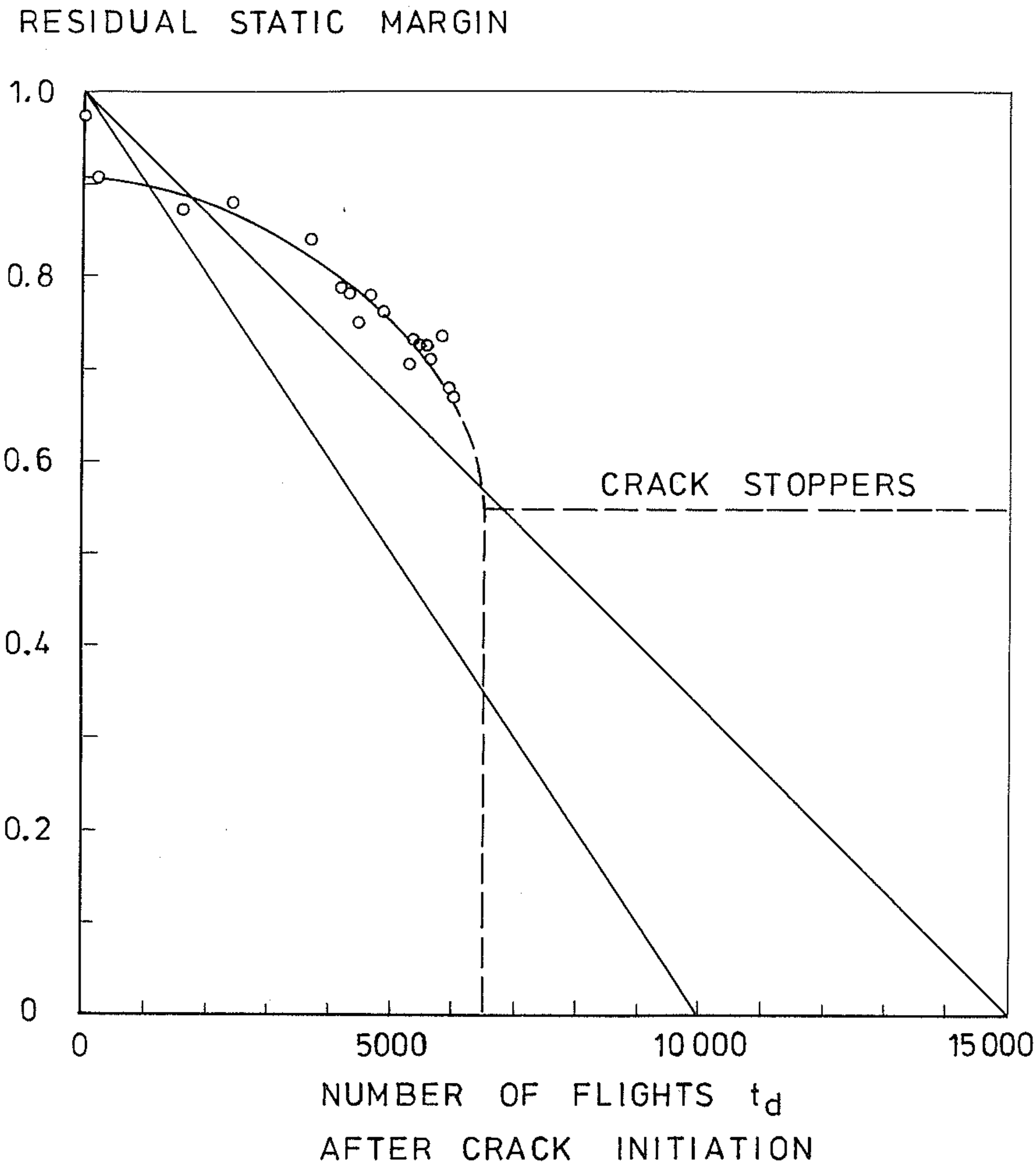

Fig. 23 Normalized residual static margin $r(t_d)$ plotted versus time t_d after crack initiation

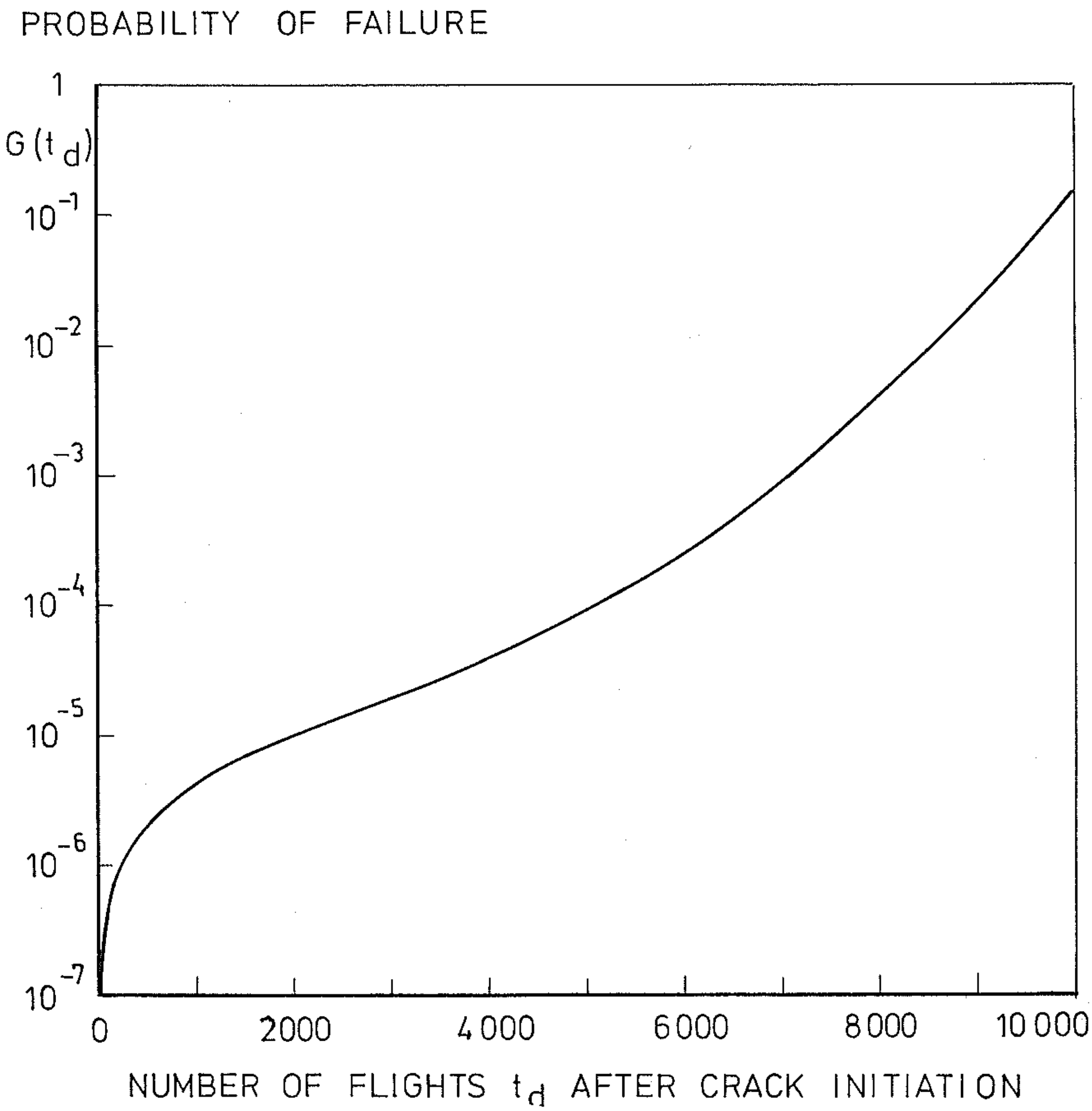

Fig. 24 Probability of failure due to a gust load in a structure with fatigue crack, versus time t_d after crack initiation

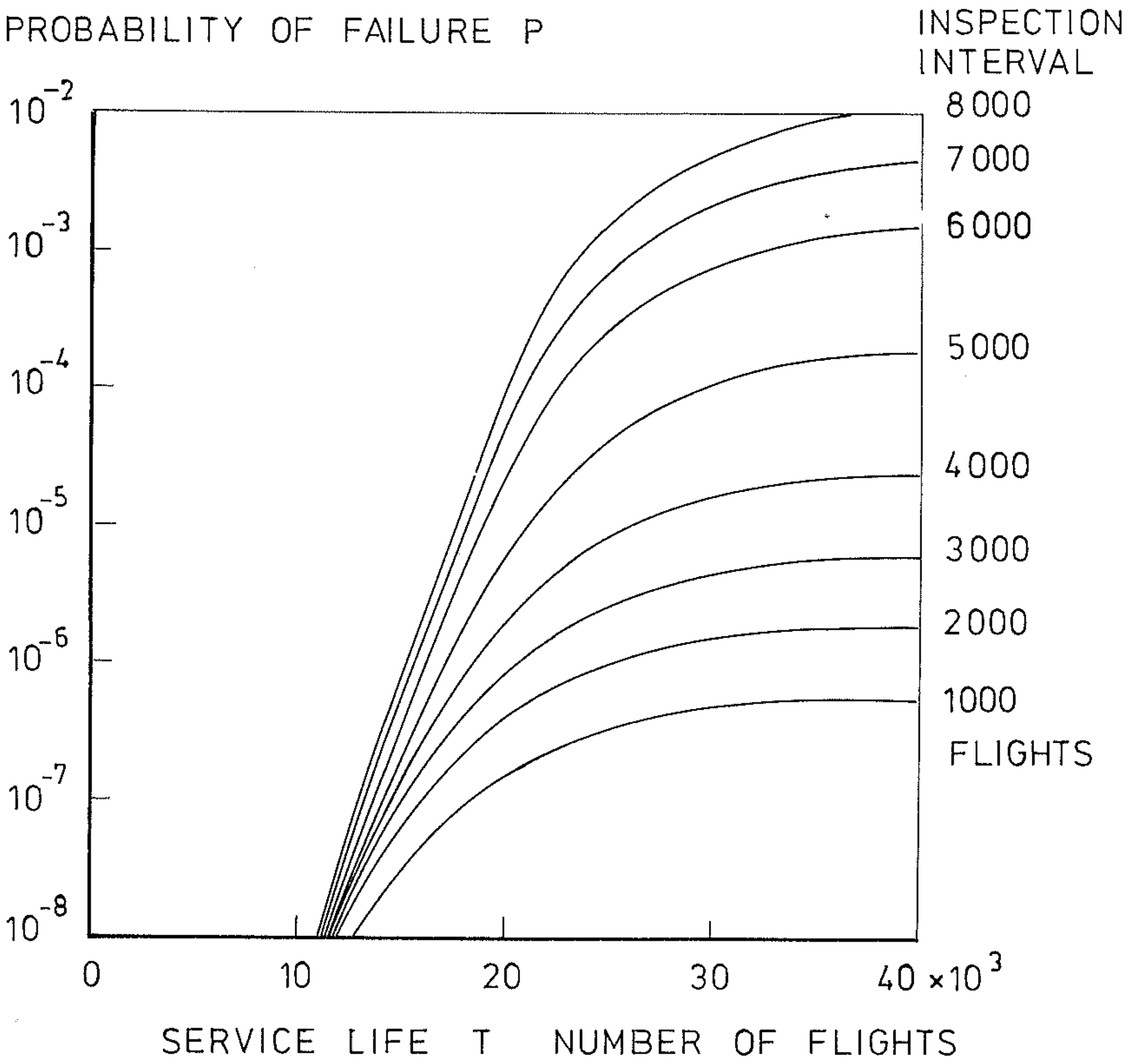

Fig. 25 Probability of fatigue failure during service life T, assuming periodic inspection with constant intervals 1000 - 8000 flights