ON THE DEVELOPMENT OF STATISTICAL KNOCK-DOWN FACTORS FOR WFD ASSESSMENT

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Abstract: The nucleation of multiple fatigue cracks over large areas, how they interact, and whether they can lead to a catastrophic event has been for a long time a concern, not only for the OEMs and the Regulatory Agencies, but also for the entire community involved in aircraft fleet management, as long as the fleets age.
The phenomenon of Widespread Fatigue Damage (WFD) has been investigated for decades, and to demonstrate freedom of WFD up to the LOV for large transport aircraft structures became mandatory in the latest amendments of the regulations.
During the ICAF Symposium in 2013, Safarian [1] has proposed an approach to assess WFD based on fatigue life estimation and the application of a set of knock-down factors. The present work provides an in-depth investigation for each of these factors, bringing a deduction or an explanation for them based on statistical approaches, together with some examples of application.
By understanding each and every factor's origin and eventual built-in conservativeness, the engineer in charge can propose alternative approaches and come out with more realistic results, eventually saving weight and resources, without compromising safety.
The present work, besides supplying the statistical basis of the knock-down factors of Ref. [1], intends to present some examples of that approach, extending its application while reducing some intrinsic conservatisms.

Keywords: WFD, statistics, reliability, knock-down factors

INTRODUCTION

Over the last 70 years, there were significant advances in aircraft structural design as design philosophies and certification requirements evolved. By the time of the Comet fatal accidents, there was a consciousness about the need of developing specific requirements related to structural integrity of aircraft during their development and operation. Besides that, it is worth to note that full-scale fatigue tests were developed for the first time and aeronautical accident investigations experienced a remarkable development at that time.

From the fifties until today, design philosophies evolved from safe life (or safety by retirement) to fail-safe (or safety by design) and then to damage tolerance (or safety by inspection). The design characteristics resulting from these philosophies (that are not excluding one from the other) may be identified in innumerous parts of the airframe. As a simple example, aircraft structures, in spite of being extremely optimized for weight saving and high performance, preserve a high degree of redundance, a basic characteristic of a fail-safe design. But as fail safety itself was recognized as incomplete, one of the main contributions added by damage tolerance philosophy during the late seventies and early eighties was the introduction of periodical structural inspections to the aircraft maintenance plan, in such a way that any occurrence either due to fatigue or manufacturing damages could be identified with sufficient anticipation by means of these inspections.

More recently, motivated mainly by an accident that occurred with a B-737 aircraft from Aloha Airlines in 1988 [2], once again there was an evolution in the fatigue and damage tolerance requirements, with the introduction of WFD and LOV, both related to aging of structures when subjected to cyclic loading for long time periods. WFD stands for "widespread fatigue damage" and LOV means "limit of validity" of engineering data for which an aircraft was designed and should operate safely. By doing a retrospective, it is noted that the WFD requirement assures structural integrity for regions where damage tolerance does not necessary work effectively, and resembles a safe life approach for fatigue.

R. D. J. Maxwell, during the ICAF Symposium of 1973 [3], about fifteen years before the Aloha accident, raised some important concerns about WFD and the LOV, stating that "it is considered that there is likely to be a finite life for fail-safe structures beyond which safety is impaired by the onset of many interacting cracks, a case which is not normally tested in current practice". Likewise, O'Brien et. al, [4], mentioned that another factor that the certification requirements were not addressing at that time was "the possibility of a number of defects developing concurrently in adjacent structure with potentially serious consequences for the fail-safe performance of the component."

It is known that fatigue is phenomenological, and that any fatigue data presents some level of scatter. As stated by Schijve [5], "the fatigue lives of similar specimens or structures under the same fatigue load can be significantly different". This subject has been studied over many decades, but some aspects seem to require further in-depth understanding. This paper presents important aspects of the statistical basis for fatigue assessment, particularly with respect to WFD. First, the fundamentals behind the methodology proposed by Safarian [1] and based on knock-down factors will be presented, followed by some examples of application, and with some comments and conclusions about this methodology.

THE KNOCK-DOWN FACTORS

In Jerusalem, 2013, Safarian [1], in light of the new requirements, presented a work bringing the concepts behind WFD and proposing a methodology for WFD assessment, from the identification of Susceptible Structures up to the definition of Dedicated Maintenance Actions.

One step of the proposed methodology deals with the estimation of when the so-called WFD average behaviour will happen, based on fatigue life estimation and the application of a set of knock-down factors which account for statistics and reliability key characteristics, as well as representativeness of the tests that serve as source data in relation to the structure under evaluation.

A total of four knock-down factors were presented by Safarian, [1], but without providing details about how they were obtained. The definitions and values of these factors were extracted from the reference and are transcribed below.

The Reliability Factor is applied to convert mean or characteristic life data into a reliable life value. Using a Weibull distribution and common shape factors, the reliability factors from Table 1 can be applied.

Table 1: Reliability Factor (source: [1]).

Material	*95% Reliability*	*99% Reliability*
Aluminium	0.48	0.32
Titanium and Steel (<200 ksi)	0.37	0.22
Steel (>200 ksi)	0.26	0.12

The Confidence Factor is a statistical-based value, which addresses the uncertainty associated with the final design value caused by the limited test sample size. For coupons and common materials, a value of 0.7 for the standard 95% lower confidence bound across the board can be used. This value is slightly conservative for aluminium and is a moderate fit for high-strength steel.

The purpose of the Scale Factor is to adjust the design life based on the ratio of details in the specimen to the number of details in the actual structure. Using Weibull statistics, the resulting factors are presented in Table 2:

Table 2: Scale Factor (source: ref [1]).

Percent of structure represented in test	Aluminium	Titanium and Steel (<200 ksi)	Steel (>200 ksi)
≥ 100	1	1	1
50-100	0.84	0.79	0.73
30-50	0.76	0.69	0.61
20-30	0.71	0.63	0.53
10-20	0.6	0.5	0.4
5-10	0.5	0.4	0.3
2-5	0.38	0.28	0.18
1-2	0.33	0.23	0.14
Less than 1%	0.28	0.19	0.1

The Testing Factor is intended to conservatively account for differences in the scale and fidelity of the test, including the extent to which the loading of the test article replicates the actual structure. This empirical factor might range from 1.0 for the fully representative test article and service loading, down to 0.7 for the small coupons tested under constant amplitude loading.

ON THE STATISTICAL NATURE OF METAL FATIGUE

As stated by Schijve [5], “the fatigue lives of similar specimens or structures under the same fatigue load can be significantly different”. Figure 1 illustrates how the scatter can be very large in a test campaign. According to Schijve [5], the scatter band can be narrower at high stress levels and wider at lower stress levels, specially near the fatigue limit this band is even wider due to the run-outs, what suggests that the scatter may be not only interpreted on the life axis, but also on the stress axis.

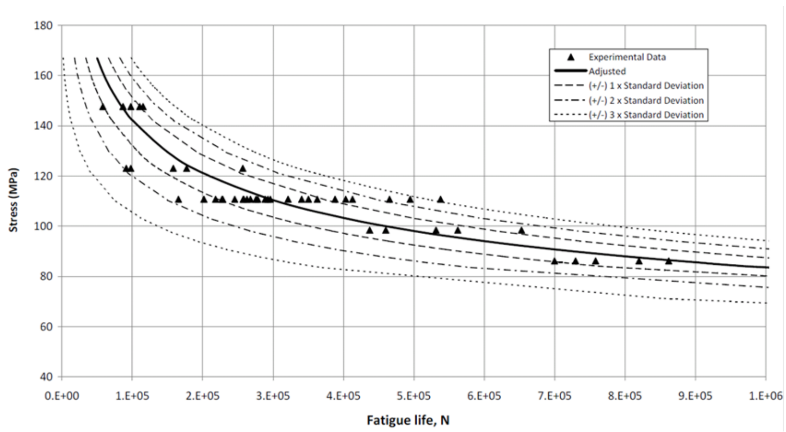


Figure 1: Scatter on fatigue test data

About the inherent spread of fatigue and the use of scatter factors, FAA AC 23-13A [6] states that the value for the standard deviation of the fatigue test life is usually taken from historical data. This is due to the high cost of testing sufficient numbers of full-scale test articles to develop an accurate value for the standard deviation. Then, Ref. [6] proposes typical values of standard deviation for various structural metals, including Aluminum alloys, Titanium alloys and Low and High Resistance Steels, i.e., a similar categorization as proposed by Safarian, Ref. [1] and presented in Table 1 and Table 2.

Whittaker and Besuner [7] presented an extensive work compiling the then available results of fatigue tests and, after a statistical treatment, concluded that "for most aluminum structural applications a reasonable value of the Weibull shape parameter is $\alpha = 4.0$ and for the log-normal case $\sigma = 0.14$".

In a further work, using a similar methodology, Whittaker [8] also investigated titanium and steel and suggested average values of the Weibull shape parameter as:

$\alpha = 3.0$ for titanium applications and for steel which has an ultimate strength < 240 ksi
$\alpha = 2.2$ for steel which has an ultimate strength > 240 ksi

THE WEIBULL DISTRIBUTION

In Nagoya, 2017, Brot [9] presented a discussion about the similarities and differences between the log-Normal and the Weibull distributions to represent the metal fatigue phenomena. The present work does not intend to go deep on this discussion, or to defend one side against the other. The Weibull model is chosen here because it is the model behind the knock-down factors from Ref [1]. However, the same deductions and approaches can be applied to the log-Normal distribution as well.

Weibull [10], formulated a statistical distribution function recognized by himself as having a wide applicability. According to O'Connor, [11], in Reliability Engineering Weibull probability data analysis is probably the most widely utilized technique of processing and interpreting life data. One of many advantages is the flexibility of the Weibull distribution, easy interpretation of the distribution parameters, and their relation to the failure rates and the bathtub curve concepts.

The simpler version of the Weibull distribution is the 2-parameter model, whose probability density function, or *pdf*, is given by Eqn. (1), and cumulative distribution function, or *cdf*, also called as Probability of Failure, is given by Eqn. (2)

$$f(x,\alpha,\beta) = \begin{cases} \frac{\alpha}{\beta}\left(\frac{x}{\beta}\right)^{\alpha-1} e^{-\left(\frac{x}{\beta}\right)^{\alpha}} & x \geq 0 \\ 0 & x < 0 \end{cases} \tag{1}$$

$$F(x,\alpha,\beta) = \begin{cases} 1 - e^{-\left(\frac{x}{\beta}\right)^{\alpha}} & x \geq 0 \\ 0 & x < 0 \end{cases} \tag{2}$$

where,

- x is the time to failure, sometimes noted also as t or N, which in general reliability applications is given in units of time as, e.g., in Flight Hours, but for aircraft fatigue is commonly given in Flight Cycles instead;
- α is the shape parameter, a dimensionless positive value, $0 < \alpha < +\infty$; variations at the value of α change the shape of the distribution, what can adapt it to represent different problems in reliability engineering;
- β is the scale parameter, given in the same units as the time to failure, being also a positive value, i.e., $0 < \beta < +\infty$, and is often called as 'characteristic life' because, for any Weibull distribution, when $x = \beta$ the cumulative distribution function will equal to 0.6321, this mean that the probability of failure equals to 63.21%, regardless of the value of α.

Notes:

a) different references use different notations for the shape and scale parameters of the Weibull distribution, and often a same greek letter is used to define different parameters according to the reference being used, what can be confusing and misleading. In this work the shape and scale parameters follow the notation used, among others, by Whittaker and Besuner [7] and by Microsoft Excel®, calling them α and β respectively.

b) as there are only time to failure higher than 0, and in this work α and β are given parameters instead of variables, from now on in this work the statistical functions presented are denoted as only function of x. Hence, Eqn. (1) and Eqn. (2) are then rewritten as Eqn. (3) and Eqn. (4)

$$f(x) = \frac{\alpha}{\beta}\left(\frac{x}{\beta}\right)^{\alpha-1} e^{-\left(\frac{x}{\beta}\right)^{\alpha}} \tag{3}$$

$$F(x) = 1 - e^{-\left(\frac{x}{\beta}\right)^{\alpha}} \tag{4}$$

Figure 2 presents some comparisons showing how the 2 parameters influence the *f(x)* and the *F(x)* in a Weibull distribution.

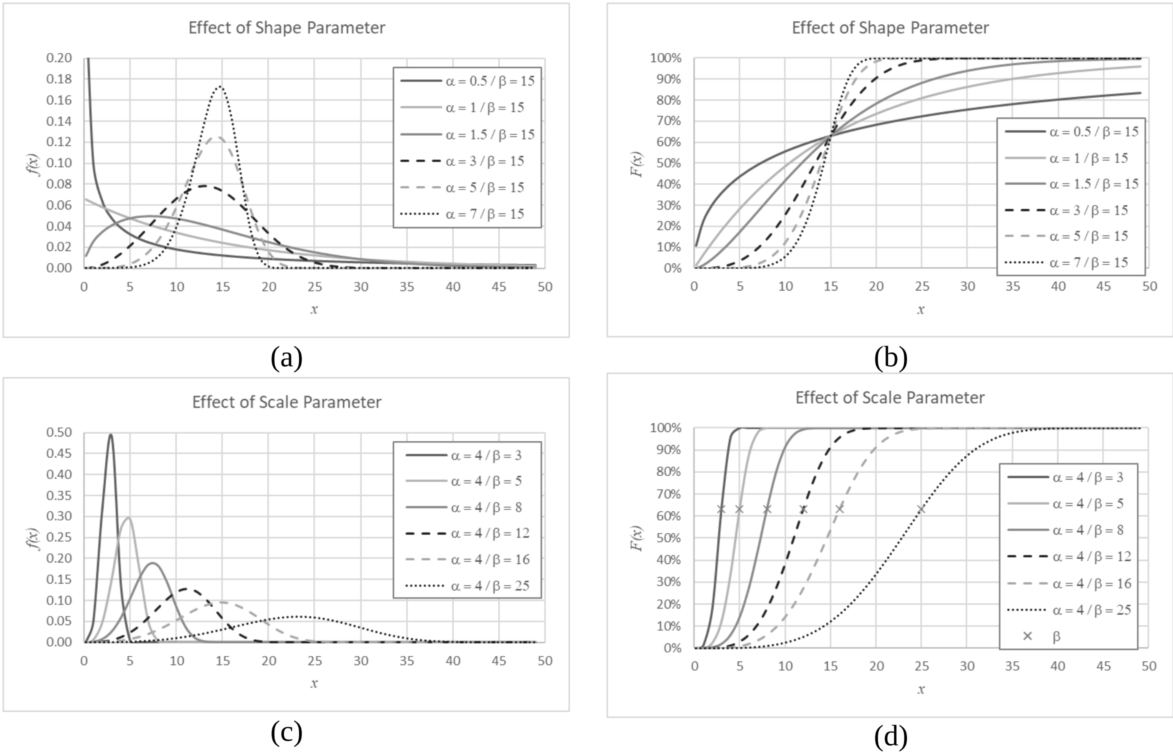


Figure 2: Influence of the 2 parameters α and β on a Weibull Distribution

Figure 2-(a) shows the effect on the probability density function when varying α while keeping β constant, where it is possible to see how flexible the Weibull distribution can be, when upon the proper choice of α the distribution can represent different engineering and physical phenomena. Still in Figure 2-(a) it is possible to see that for values of α in the range typically used for metal fatigue (see the curves of α between 3 and 7 in the figure), the distribution assumes a 'bell-like' shape, and the higher the shape parameter the lower is the spread of the distribution.

Figure 2-(b) presents a comparison of the probability of failure, still under the variation of α. The first aspect that can be verified is the representation of β as the 'characteristic life', being the point where all curves crosses, regardless of α. When $x = \beta$ the probability of failure equals to 63.21%. Another conclusion is that the higher the α the steeper is the curve, meanig less spread on the distribution.

Figure 2-(c) and Figure 2-(d) show similar comparisons of *f(x)* and *F(x)* when varying β while keeping α constant. The value of α is choosen as equal to 4.0, and Figure 2-(c) shows that the 'bell-like' shape

is attained, regardless of β. On Figure 2-(d) the probability of failure of 63.21% is highlighted for each curve, evidencing how it follows the variation on the characteristic life ($x = \beta$).

In Reliability Engineering, the Probability of Failure $F(x)$ is often also called Unreliability, being the opposite of the Reliability, $R(x)$, or the probability of an item to survive at a given time. Therefore, at any given time, when adding the Reliability and the Unreliabilty one will always find a total of 100%. Hence, the reliability function, $R(x)$, for the 2-parameter Weibull distribution is then given by Eqn. (5)

$$R(x) = e^{-\left(\frac{x}{\beta}\right)^{\alpha}} \tag{5}$$

DEVELOPING THE KNOCK-DOWN FACTORS STATISTICALLY

The Reliability Factor

As mentioned above, Reliability is the opposite of the Probability of Failure, being then the probability of an item or component to survive at a given time. The Reliability Factor aims to convert characteristic life (β) into a reliable life value.

The equation of the Reliability, Eqn. (5), can be manipulated in order to obtain a fraction the characteristic life that corresponds to a given reliability when the shape parameter is known. Applying the natural logarithm to both sides to eliminate the exponential from Eqn. (5) results in Eqn. (6), which can be rearranged as Eqn. (7) and then both sides are rised to the power of $1/\alpha$, isolating at the left side only the fraction x/β, which is the concept of the Reliability Factor itself, as shown in Eqn. (8).

$$ln[R(x)] = \left[-\left(\frac{x}{\beta}\right)^{\alpha}\right] \tag{6}$$

$$\left(\frac{x}{\beta}\right)^{\alpha} = -ln(R_x) \tag{7}$$

$$Reliability\ Factor = \frac{x}{\beta} = [-ln(R_x)]^{1/\alpha} \tag{8}$$

Using the typical shape parameters from Whittaker and Besuner [7] and Whittaker [8] in Eqn. (8) and the Reliability levels of 95% and 99%, the Table 1 can be fully reproduced, as can be seen on Table 3.

Table 3: Reliability Factor

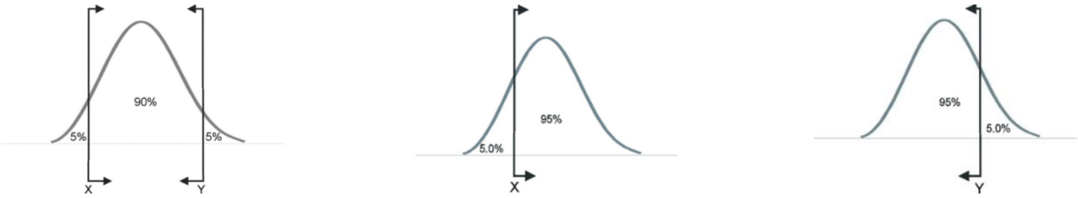
Material	α	*95% Reliability*	*99% Reliability*
Aluminium	4.0	0.476	0.317
Titanium and Steel (<200 ksi)	3.0	0.372	0.216
Steel (>200 ksi)	2.2	0.259	0.124

The Confidence Factor

O'Connor [11], introduces the concept of *degree of belief*, and compares the situations where small batches of components are inspected looking in a production line for defects and different outcomes are observed, but when the historic of several batches is accounted for, the number of samples is higher and the degree of belief on the results also become higher, concluding that the assertions that one can make based on sample statistics can be made with a degree of confidence which depends upon the size of the sample.

According to Ref. [12], if a test is repeated several times, when analyzing the results one can obtain slightly different parameters for the distribution each time, leading to slightly different reliability results. However, by employing confidence bounds, it is possible to obtain a range within which these reliability values are likely to occur a certain percentage of the time.

Confidence bounds may be one-sided or two-sided, as depicted in Figure 3. When one is interested in the the variation around the mean values, as for example in election polls margins of error, then the two-sided represented in Figure 3-(a) is commonly used. For fatigue problems, the high life outliers usually are not a concern, while one is focused on assuring a minimum life free of cracks, then the lower confidence bounds, depicted in Figure 3-(b) are of interest. Examples of application of upper confidence bounds, as represented in Figure 3-(c), may be the probability of detecting a crack using a given inspection method, where the concern is to assure a low probability of not detecting a crack larger than a given size.



(a) (b) (c)

Figure 3: Confidence Bounds

According to Whittaker and Besuner, [7], the exact lower confidence bound for a Weibull Scale Parameter can be determined by Eqn. (9).

$$\breve{\beta}_\gamma = \hat{\beta}\left[\frac{2n_f}{\chi^2_{\gamma(2n_f)}}\right]^{1/\alpha} \quad (9)$$

where:

α is the shape parameter for the detail or component under evaluation,
γ is the desired confidence level, typically 95%,
$\hat{\beta}$ is the estimated scale parameter, without considering confidence bounds,
$\breve{\beta}_\gamma$ is the scale parameter for the Lower Bound at the desired confidence level,
n_f is the number of failure observations in the test sample,
$\chi^2_{\gamma(2n)}$ is the γ-fractile of the chi-squared variate with $2n$ degrees of freedom.

As the Confidence Factor is the ratio between the lower confidence bound to the median time-to-failure value, as represented on Eqn. (10).

$$Confidence\ Factor = \frac{\breve{x}_\gamma}{x} = \frac{\breve{\beta}_\gamma[-ln(R_x)]^{1/\alpha}}{\hat{\beta}[-ln(R_x)]^{1/\alpha}} = \frac{\breve{\beta}_\gamma}{\hat{\beta}} \quad (10)$$

For a constant shape parameter and at any given reliability level from Eqn. (9) and Eqn. (10), the confidence factor can be defined as Eqn. (11).

$$Confidence\ Factor = \frac{\breve{\beta}_\gamma}{\hat{\beta}} = \left[\frac{2n_f}{\chi^2_{\gamma(2n_f)}}\right]^{1/\alpha} \quad (11)$$

Using Microsoft Excel® the chi-squared parameter can be obtained using the function *CHISQ.INV[γ;$2n_f$]*. For 95% lower confidence bound ($\gamma = 0.95$) and a single failed sample ($n_f = 1$), Eqn. (12) provides the denominator of Eqn. (11), and the Confidence Factor can then be obtained for the typical Weibull shape paramaters of the different materials, as presented on Table 4.

$$\chi^2_{\gamma(2n_f)} = \chi^2_{0.95(2\times1)} = CHISQ.INV[0.95; 2\times 1] = 5.9915 \quad (12)$$

Table 4: Confidence Factor

Material	α	*Confidence Factor*
Aluminium	4.0	0.76
Titanium and Steel (<200 ksi)	3.0	0.69
Steel (>200 ksi)	2.2	0.61

Confidence factor values calculated at Table 4 are close to the value of 0.7 introduced by Safarian, [1].

The Reliability Level, considering the Lower Confidence Bound, for any given time, is then given by Eqn. (13).

$$R_{\gamma}(x) = exp\left[-\left(x/\breve{\beta}_{\gamma}\right)^{\alpha}\right] \tag{13}$$

The Scale Factor

The Scale Factor as defined by Safarian, [1], is derived from a hypothesis that considers the fatigue failure of the first detail as the failure of the complete structure. Crack growth, crack interactions, load redistribution, among other effects on the actual structure are not accounted for, what evidences the conservativeness of such approach.

These hypothesis and simplifications can be compared to a chain, whose reliability equals to the product of the individual reliability of its n links, as per Eqn. (14).

$$R(x)_{chain} = \prod_{i=1}^{n}\left[R(x)_{link\_i}\right] \tag{14}$$

Considering that all individuals present the same characteristic life, and hence similar reliability at a given time (x), then the reliability of the chain turns into Eqn. (15).

$$R(x)_{chain} = \left(R(x)_{link}\right)^{n} \tag{15}$$

The reliability of a single link is given by Eqn. (5), reproduced here as Eqn. (16) for the particular link.

$$R(x)_{link} = e^{-\left(\frac{x}{\beta_{link}}\right)^{\alpha}} \tag{16}$$

By replacing the individual link reliability from Eqn. (16) into the reliability of the chain from Eqn. (15), it then becomes Eqn. (17).

$$R(x)_{chain} = e^{-n.\left(\frac{x}{\beta_{link}}\right)^{\alpha}} \tag{17}$$

Manipulating Eqn. (17) it can be re-written as the chain life as a result of the links characteristics, at Eqn. (22).

$$\ln[R(x)_{chain}] = \ln\left[e^{-n.\left(\frac{x}{\beta_{link}}\right)^{\alpha}}\right] \tag{18}$$

$$-n.\left(\frac{x}{\beta_{link}}\right)^{\alpha} = \ln[R(x)_{chain}] \tag{19}$$

$$\ln\left(\frac{x}{\beta_{link}}\right)^{\alpha} = \ln\left[-\frac{\ln[R(x)_{chain}]}{n}\right] \tag{20}$$

$$\ln\left(\frac{x}{\beta_{link}}\right) = \frac{1}{\alpha}.\ln\left[-\frac{\ln[R(x)_{chain}]}{n}\right] \tag{21}$$

$$x_{chain} = \beta_{link}.\exp\left\{\frac{1}{\alpha}.\ln\left[-\frac{\ln[R(x)_{chain}]}{n}\right]\right\} \tag{22}$$

The same manipulation, when applied to reliability of a single link from Eqn. (16) gives Eqn. (23)

$$x_{link} = \beta_{link}.\exp\left\{\frac{1}{\alpha}.\ln[-\ln[R(x)_{link}]]\right\} \tag{23}$$

The Scale Factor can be defined as the ratio between the actual structure life (the chain in this example) to the tested structural detail (represented by the individual links in this example) for the same reliability level, $R_{chain} = R_{link} = R$, and is obtained combining and manipulating Eqn. (22) and Eqn. (23).

$$Scale\ Factor = \frac{x_{chain}}{x_{link}} = \frac{\beta_{link}.\exp\left\{\frac{1}{\alpha}.\ln\left[-\frac{\ln(R)}{n}\right]\right\}}{\beta_{link}.\exp\left\{\frac{1}{\alpha}.\ln[-\ln(R)]\right\}} \tag{24}$$

$$Scale\ Factor = \exp\left[\left\{\frac{1}{\alpha}.\ln\left[-\frac{\ln(R)}{n}\right]\right\} - \left\{\frac{1}{\alpha}.\ln[-\ln(R)]\right\}\right] \tag{25}$$

$$Scale\ Factor = \exp\left[\left\{\frac{1}{\alpha}.\ln[-\ln(R)]\right\} + \left\{\frac{1}{\alpha}.\ln[n^{-1}]\right\} - \left\{\frac{1}{\alpha}.\ln[-\ln(R)]\right\}\right] \tag{26}$$

The two terms on R cancel eachother and finally the Scale Factor is obtained by Eqn. (27).

$$Scale\ Factor = \exp\left(-\frac{1}{\alpha}.\ln[n]\right) \tag{27}$$

Some conclusions can be drawn from Eqn. (27): the Scale Factor does not depend on the Characteristic Life β nor on the Reliability Level R, it depends only on the Shape Parameter α and the number of times n that the elementary detail is replicated on the actual structure.

In order to validate Eqn. (27) and the "link versus chain" approach, the Scale Factor values from in Table 2 are compared to the Scale Factors calculated through Eqn. (27). This comparison is presented on Figure 4, validating the presented approach.

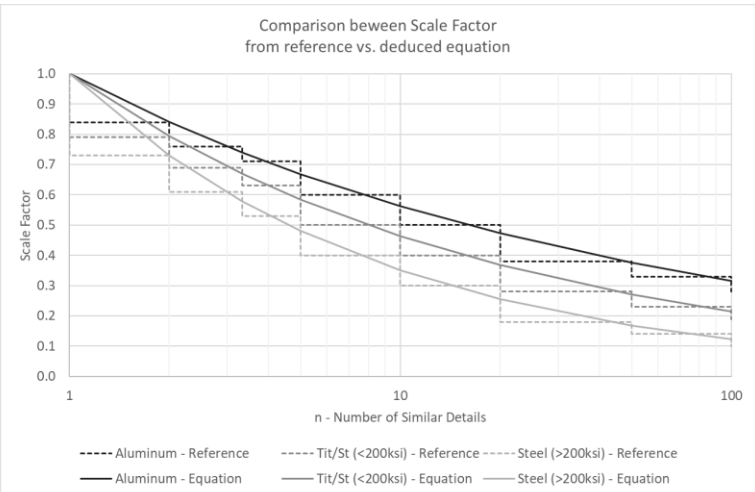


Figure 4: Validation of the Scale Factor Equation

It is important to notice that the Scale Factor described above is derived from the assumption that all elementary components from one assembly have the same reliability level at a given time.

If the elementary components have different characteristic lives, leading to different reliability levels at the same time, then the Scale Factor cannot be used, and the product of the individual reliabilities, as per Eqn. (14), shall be used instead, accounting for such differences.

The Testing Factor

Among the four factors introduced by Safarian [1], the Testing Factor is the only one that has not a statistical origin. Rather, it is said to have an empirical origin and is supposed to account for differences between the test and the actual structure, including the representativeness of the in-service loading.

Schijve, [5], when addressing Fatigue under Variable Amplitude Loading and the Pålmgren-Miner rule, mentions that numerous VA fatigue test programs were conducted, and in many cases significant discrepancies were found. Schijve then discusses some inconsistencies of the rule and outlines that certain shortcomings of the rule must be understood.

About the abovementioned numerous variable amplitude fatigue test programs, Schijve [5] mentions the work from Schütz [13] and presents a figure comparing the actual test results to the life predictions for different variable amplitude fatigue test programs. The mentioned figure is adapted here as Figure 5.

Two groups of tests are compared on Figure 5, namely 29 test series under load sequences with a constant mean stress and a random variation of the load amplitude and 57 test series under load sequences with large variations of the mean stress in addition to amplitude changes. Despite the scatter on the results, the first group presents a good match for the mean value across the 29 series, while the later showed that the Miner rule may lead to unconservative results, with the mean value of the 57 series resulting in values around 0.6.

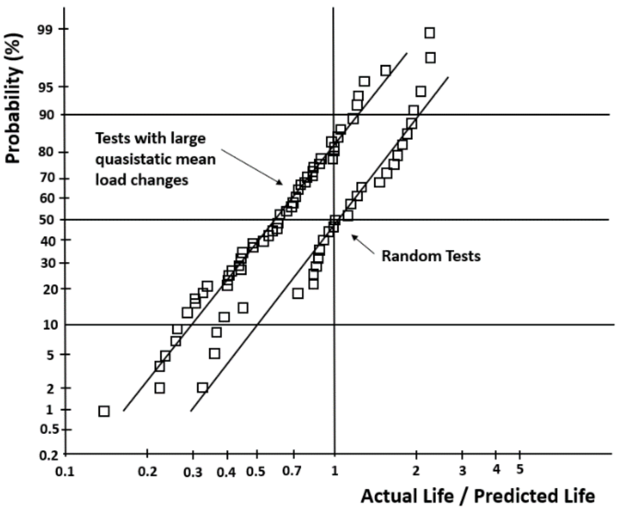


Figure 5: Comparison between test results and life predictions in VA fatigue tests. Adapted from [5]

Schijve [5] and Schütz [13] present the concept of a Relative Miner Rule, where, given the unconservative predictions of the Miner rule, the classical $\sum n/N = 1$ is replaced by $\sum n/N = q$, with $q < 1$. Still according to Schijve, [5], the value of q has to be selected by experience of variable amplitude tests with similar load-time histories relevant to the problem under consideration.

The authors of the present work believe that the value of the testing factor equal to 0.7 for small coupons tested under constant amplitude loading to be used for variable amplitude applications proposed by Safarian [1] is a correction to the Miner rule, and may have the same meaning of the q proposed by

Schijve [5] at his Relative Miner Rule. The present work does not intend to draw additional or final conclusions on this factor, but recommends that such shortcomings of the Miner rule have to be further investigated and better understood.

EXAMPLES AND DISCUSSION

The Typical Scatter Factors for Fatigue

The factors introduced by Safarian [1] can also be applied to the fatigue evaluation of single details, rather than only to structures with repetitive design details which are susceptible to WFD. In this case, a single detail on the structure would correspond to the small aluminum specimens tested under constant amplitude load, and the Scale Factor equals to 1.0. For an application subjected to variable amplitude loading the Testing Factor equals to 0.7. For a reliability target of 95% with a confidence level of 95%, the Reliability Factor is 0.48, while the Confidence Factor is 0.7. The $N_{95/95}$ life is then obtained by multiplying these 4 factors over the characteristic life, as per Eqn. (28).

$$N_{95/95} = RF \cdot CF \cdot SF \cdot TF \cdot \beta = 0.48 \cdot 0.7 \cdot 1.0 \cdot 0.7 \cdot \beta = 0.24\beta \tag{28}$$

Fatigue properties are usually given in terms of mean values, or N_{50} instead of characteristic life (β). Using Eqn. (8) it is possible to convert characteristic into mean lives by obtaining the Reliability Factor for 50% Reliability

$$RF_{50} = N_{50}/\beta = [-ln(0.5)]^{1/4} = 0.912 \tag{29}$$

$$N_{50} = 0.912\beta \rightarrow \beta = N_{50}/0.912 \tag{30}$$

Combining Eqn. (30) and Eqn. (28) gives

$$N_{95/95} = (0.24/0.912).N_{50} \approx N_{50}/4 \tag{31}$$

From Eqn. (31) it can be seen that the typical scatter factor of 4.0, widely used on fatigue applications, transforms mean values into life data with 95% reliability and 95% confidence.

Eqn. (28) accounts for differences in the test loading comparing to the actual structure, once the testing factor of 0.7, which corresponds to small coupons tested under constant amplitude loading, is applied. However, if the actual structure is also subjected to constant amplitude loading only (e.g. a pressure bulkhead, not subjected to spectrum loading), or the fatigue data come from test subjected to a spectrum load representative of the real usage, the testing factor becomes 1.0, and the scatter factor can be reduced down to 3.0.

Calculating an Intermediate Scale Factor

The equation for the Scale Factor deduced above, namely Eqn. (27), can replace the factors from Table 2 for intermediate values.

An engineer has to evaluate an aluminum structure that has repetitive design details, totalizing 6 times more features than the reference arrangement which was tested and whose results are available. A Scale Factor has then to be applied. The test represents 1/6, or 16.7% of the structure. Examining Table 2 it can be seen that there is a range from 10% to 20%, what gives a scale factor of 0.6.

On the other hand, by applying Eqn. (27) the engineer concludes that a factor of 0.64 can be used instead, a difference higher than 5% in comparison to the value from Table 2.

This example leads to the conclusion that the equation for the Scale Factor can provide more precise values than the tabulated ones.

Calculating the Knock Down Factors for a Different Alfa

The work of Whittaker and Besuner, Ref [7], lists at its appendices the fatigue results of every single test article belonging to the several groups of tests from the data collection used on the development of the work.

One of these groups of tests was chosen for this example, namely Item 417, composed of 31 test articles which, according to the coding of the data, belong to test # 09010865080, corresponding to Aluminum 7075-T6, 0.09” thick, notched, with normal finishing, spectrum loaded, with specimens cycling up to their complete failure. Table 5 presents the number of cycles up to the failure of each single specimen from Item 417, while Table 6 summarizes the parameters estimated by Whittaker and Besuner [7] for both distributions, Log-normal and Weibull.

Table 5: Number of cycles to failure for the data from Item 417 of Whittaker and Besuner, [7]

30,926	34,554	36,381	38,423	40,103	40,501	42,280	44,392
46,092	46,125	46,125	48,025	48,025	48,055	48,055	48,055
48,055	48,056	51,675	52,344	52,345	52,345	52,345	52,379
55,997	56,202	57,709	57,709	57,709	57,709	63,494	

Table 6: Parameters estimated for Item 417

Log-normal		Weibull	
Average	Std Dev	β	α
47,845	0.0719	51,589	7.4119

Suppose that an engineer has to define the number of cycles with 99% reliability and 95% confidence for a structure made out of the same material, subjected to variable amplitude loading and which contains 20 repetitive design details.

Initially the engineer goes to a reference database as, say, the MMPDS, obtains the mean or characteristic life and applies the set of factors from [1]. The Reliability factor is 0.32, the Confidence factor is 0.7, the Testing factor also is 0.7, and, for 20 details, or 5% of the structure represented in the test, the Scale factor is 0.5. The knock-down factor is then obtained by Eqn. (32)

$$knock\ down|_1 = RF * CF * TF * SF = 0.32 * 0.7 * 0.7 * 0.5 = 0.078 \quad (32)$$

Supposing that after realizing that this factor does not meet the project requirements, a test campaign is conducted, and the results from Table 5 are obtained. Then the knock down factors are calculated based on the results from Table 6, or $\alpha = 7.4119$.

Based on Eqn. (8) the Reliability Factor is calculated as shown in Eqn. (33)

$$Reliability\ Factor = x/\beta = [-ln(R_x)]^{1/\alpha} = [-ln(0.99)]^{1/7.4119} = 0.54 \quad (33)$$

The Confidence Factor is obtained applying Eqn. (11), and the result is shown on Eqn. (34).

$$Confidence\ Factor = \left[\frac{2n_f}{\chi^2_{\gamma(2n_f)}}\right]^{1/\alpha} = \left[\frac{2 \times 31}{\chi^2_{0.95(2\times 31)}}\right]^{1/7.4119} = \left[\frac{62}{81.38}\right]^{1/7.4119} = 0.96 \quad (34)$$

The Scale Factor is obtained by Eqn. (27), resulting in Eqn. (35).

$$Scale\ Factor = \exp\left(-\frac{1}{\alpha}.\ln[n]\right) = \exp\left(-\frac{1}{7.4119}.\ln[20]\right) = 0.67 \quad (35)$$

Regarding the Testing Factor, as the data from Table 5 were obtained from specimens subjected to a spectrum load representative of the service loads, then the Testing Factor equals to 1.0.

The new knock-down factor is then obtained by Eqn. (36).

$$knock\ down|_2 = RF * CF * TF * SF = 0.54 * 0.96 * 1.0 * 0.67 = 0.35 \quad (36)$$

Comparing the resulting factors from Eqn. (32) and Eqn. (36) it is possible to note that the test data, and the factors obtained from them, can lead to life results more than 4 times better than if no test were available and the reference factors would be used instead, leading to the conclusion that for many particular cases it is worth investing on developing tests and building their own data bank.

Obtaining the Knock-down Factors for Different Distributions

Eqn. (15) represents the reliability of a chain as the product of the reliability of its individual links provided that all individuals present similar characteristic lives leading to similar reliability levels at any given time. This can be generalized for the reliability of an assembly of similar parts or similar details, as per Eqn (37), where the subscripts were adapted for the assembly of *n* similar details. But usually the project target is the reliability of the assembly and for achieving it a minimum reliability of the individual details is of interest during the design evaluation, rewriting Eqn (37) as the individual reliability as a function of the assembly target reliability, as shown in Eqn (38).

$$R(x)_{assembly} = (R(x)_{detail})^n \quad (37)$$

$$R(x)_{detail} = \left(R(x)_{assembly}\right)^{\frac{1}{n}} \quad (38)$$

Eqn (38) may be applied to replace, at the same time, the Reliability and the Scale Factors. For the Weibull distribution this combined factor may be obtained by combining Eqn (38) and Eqn (8), becoming Eqn (39).

$$Combined\ Factor|_{RF+SF} = \frac{x}{\beta} = \left[-ln\left(\left(R(x)_{assembly}\right)^{\frac{1}{n}}\right)\right]^{1/\alpha} \quad (39)$$

For an assembly of 100 similar details, with a target assembly reliability of 99% and $\alpha = 4$ for aluminum, the combined factor is calculated at Eqn (40)

$$Combined\ Factor|_{RF+SF} = \frac{x}{\beta} = \left[-ln\left((0.99)^{\frac{1}{100}}\right)\right]^{1/4} = 0.10 \quad (40)$$

In order to verify Eqn (39), Eqn. (8) and Eqn. (27) are combined, and the resulting value is compared with Eqn (40), as it can be seen on Eqn. (41), leading to the conclusion that Eqn (38) can be used as a general rule to account, at the same time, for the reliability and scale factors.

$$RF \times SF = \left([-ln(0.99)]^{1/4}\right) \times (\exp(-(1/4).\ln[100])) = 0.317 \times 0.316 = 0.10 \quad (41)$$

In the example above, what would be the combined factor if a log-Normal distribution is being used instead of Weibull?

One of the biggest advantages of the Weibull distribution, when compared to other distributions widely used, is its simplicity and ease of mathematical manipulation. The Normal and the log-Normal distributions can only be integrated numerically, what can be done with the aid of a software or with tabulated values. In Microsoft Excel® the function *NORM.INV[p;μ;σ]* returns the inverse of the normal cumulative distribution for the specified probability level (*p*), mean (μ) and standard deviation (σ).

In the example above, the target reliability of the single detail from Eqn (38) would be $0.99^{\left(\frac{1}{100}\right)}$, or 99.99%. Supposing that the detail would have a mean fatigue life of 10^6 cycles, and, from Whittaker and Besuner [7], the typical standard deviation for aluminum being $\sigma = 0.14$, the inverse of the Normal cumulative distribution is presented in Eqn. (42), but as the distribution is the log-Normal, this result has to be elevated to the power of 10 to convert into life results. Eqn. (43) presents then the equivalent of the Combined Factor, which accounts for the Reliability and the Scale Factors.

$$\log x|_{assembly,R99\%} = NORM.INV[(1-0.9999); \log 10^6; 0.14] = 5.4795 \quad (42)$$

$$Combined\ Factor|_{RF+SF} = \frac{x_{assembly,99\%}}{\mu} = \frac{10^{5.4795}}{10^6} = \frac{301{,}657}{1{,}000{,}000} = 0.3 \quad (43)$$

Comparing the results from Eqn. (43) to Eqn. (41) one can conclude that the Weibull distribution leads to factors 3 times more severe than the log-Normal distribution in this example.

Splitting the effects, the equivalent of the reliability factor for 99% reliability level can be obtained repeating the steps from Eqn. (42) and Eqn. (43), but considering the pure 99% reliability, without accounting for the repeated details, as presented in Eqn. (44) and Eqn. (45), after that, the results from Eqn. (43) can be divided by the results of Eqn. (45) resulting in the equivalent of the Scale Factor, which is presented on Eqn. (46).

$$\log x|_{R99\%} = NORM.INV[(1-0.99); 6; 0.14] = 5.6743 \quad (44)$$

$$Reliability\ Factor = \frac{x_{99\%}}{\mu} = \frac{10^{5.6743}}{10^6} = \frac{472{,}401}{1{,}000{,}000} = 0.472 \quad (45)$$

$$Scale\ Factor = \frac{Reliability\ Factor \times Scale\ Factor}{Reliability\ Factor} = \frac{0.3}{0.472} = 0.639 \quad (46)$$

These results are summarised on Table 7.

Table 7: Comparison between Weibull and log-Normal

	Weibull	log-Normal	log-N/Weibull
Reliability (99%)	0.317	0.472	1.5
Scale (n = 100)	0.316	0.639	2.0
Combined	0.10	0.30	3.0

A broad discussion about the differences of the Weibull and log-Normal distributions for representing metal fatigue behaviour was brought by Brot, Ref. [9], and corroborates the differences evidenced in Table 7. The present work does not intend to go deeper in this discussion. Rather, the proposal here is to illustrate that, once the statistical origin of the knock-down factors is known, their concepts can be applied to other distributions as well, but sometimes it has to be made using numerical results instead of analytical equations as for Weibull.

CONCLUSIONS

This work brought an in-depth investigation for the factors from Ref. [1], presenting a background and deduction or an explanation for them based on statistical approaches.

The Reliability and the Confidence Factors convert the mean or average values into values that meet the lower probability of failure or higher reliability targets, at the same time that accounts for the uncertainties due to the reduced test samples.

The Testing Factor has no statistical origin, being more related to the shortcomings of the Pålmgren-Miner rule, and has to be further investigated and understood.

The Scale Factor has a similar effect of requiring higher reliability levels, and may be either optimistic or overconservative based on the statistical distribution chosen. In addition, it is derived from the assumption that the failure of an entire assembly occurs when its first individual element presents a crack nucleation, what is also conservative because it does not account for crack growth, crack interactions, load redistribution, among other effects on the actual structure. Such conservativeness can be further removed by testing structural arrangements as close as possible the actual structural arrangement.

By understanding each and every factor's origin and built-in conservativeness, the engineer in charge can propose alternative approaches and come out with more realistic results, eventually saving weight and resources, and developing optimized structures without compromising safety.

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