CAN WE PREDICT FATIGUE CRACK GROWTH WITHOUT THE HELP OF PARIS?

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Abstract: Prediction models for fatigue crack growth in metallic materials utilize so-called 'Paris laws' to predict how cracks develop under cyclic loading. Albeit very successful in engineering practice, this practice constitutes *interpolations* instead of *predictions*, considering that always first sufficient number of fatigue crack growth tests must be performed through which these phenomenological relations are fitted.
To really predict fatigue crack growth, i.e. without pre-existing fatigue crack growth data, requires a better understanding of the physics of fatigue crack growth and of the role plasticity has on this phenomenon. To that aim, this paper explains the essential steps necessary to describe the physics of fatigue loading (application of strain energy or work) and the corresponding dissipation of energy through the formation of fracture surfaces and plastic volume around the crack tip. This method allows to generate as prediction *result* the Paris curve using stress-strain data, rather than using it as *input* in prediction.

Keywords: Fatigue crack growth, physics, predictions, strain energy dissipation

INTRODUCTION

Prediction or interpolation?

Since the introduction of the often so-called 'Paris law', named after Paul Paris [1], many engineers and researchers have successfully predicted fatigue crack growth in metallic structures utilizing the Paris curves. After this introduction, various versions of the Paris relationship have been proposed and used, of which the NASGRO relation [2] and similarly the Hartman-Schijve relation [3] are famous examples. One can debate, however, whether this practice really constitutes *predictions*, considering that always first sufficient number of fatigue crack growth tests must be performed, through which these phenomenological relations are fitted. Essentially, these predictions constitute *interpolations* between pre-existing fatigue crack growth data, which are in most cases calibrated and validated with the same test data used to generate the input parameters.
To really predict fatigue crack growth, i.e. without pre-existing fatigue crack growth data, requires a better understanding of the physics of fatigue crack growth and of the role plasticity has on this phenomenon. However, the current body of empirical and phenomenological work does not deliver for such physics based description. This paper aims to demonstrate that it is possible to predict fatigue crack growth, without using pre-existing fatigue crack growth data. Particularly, the hypothesis underneath this work is that the quasi-static mechanical properties provide sufficient information to describe the low strain rate material behaviour under cyclic loading.

PHYSICAL ENERGY BALANCE

According to Irwin and Orowan [4] who studied static crack growth, the crack growth resistance equals the sum of plastic strain energy and the surface energy due to crack growth. Considering the conservation of energy, the external work done to the cracked plate by the applied force is considered here the energy input. Part of this energy is stored within the plate as elastic strain energy, while the remaining is dissipated through plasticity and fracture. Therefore the energy conservation can be formulated as [5]

$$\dot{W} = \dot{U}_a + \dot{U}_p + \dot{U}_e \tag{1}$$

where the differential form of the equation describes that any change in the work applied, is equal to the sum of the three terns: energy dissipation U_a and U_p and the change in energy stored U_e. Keep in mind that this equation applies to any moment or increment in a load cycle. Hence, the differential expression is interpreted as $\dot{W} = dW/dt$, which can be translated through full integration over a single load cycle to $f \cdot dW/dN$ with the frequency f defined as $f = N/t$.

DESCRIPTION OF METHODOLOGY

Applied work

Uniaxially loading an uncracked (a = 0) plate by a stress S, implies the application of work (or strain energy) to that plate's volume

$$U_0 = \frac{1}{2} S\varepsilon WLt = \frac{1}{2}\frac{S^2}{E} WLt \tag{2}$$

where W, L, and t are respectively the plate with, length and thickness, and E is the material's Young's modulus. While cyclic loading in fatigue crack growth models is generally described by the stress range ΔS or stress intensity factor range ΔK or ΔK_{eff}, this cyclic loading relates to cyclic work through

$$\Delta U_0 = U_{0,max} - U_{0,min} = \frac{1}{2}\frac{S_{max}^2 - S_{min}^2}{E} WLt \tag{3}$$

when a crack is propagating in the plate, then the plate compliance is increasing, equivalent to a stiffness decrease, which in load controlled conditions implies that more work is applied to the plate. The finite width correction factor accounts for this effect of compliance increase on the work applied through [6]

$$\Delta U_a = U_{a,max} - U_{a,min} = \frac{1}{2}\frac{S_{max}^2 - S_{min}^2}{E} WLt \sqrt{sec\left(\frac{\pi a}{W}\right)} \tag{4}$$

Note that at S_{max}, the total elastic energy applied to the plate is described by

$$U_{a,max} = \frac{1}{2}\frac{S_{max}^2}{E} WLt \sqrt{sec\left(\frac{\pi a}{W}\right)} \tag{5}$$

Fracture surface energy dissipation

When the crack propagates, new fracture surfaces are created. Creating these fracture surfaces, requires the dissipation of strain energy, which can be described per load cycle by

$$\frac{dU_a}{dN} = \frac{dU_a}{dA} t\gamma\lambda \frac{da}{dN} \tag{6}$$

Here, the surface energy dU_a/dA is the energy required to create the crack of area A, representing the difference between internal energy within the atom lattice between the centre of a unit cell and its surface. For aluminium the typical value is about 1 J/m^2, which yields 2 J/m^2, in this case, as two fracture surfaces are formed during crack growth. The two factors λ and γ are introduced here, to account for the fact that the true area of the fracture is influenced by respectively the shear lip formation and the fracture surface roughness, relative to the projected planar fracture area $A = t\,a$, see Figure *1*.

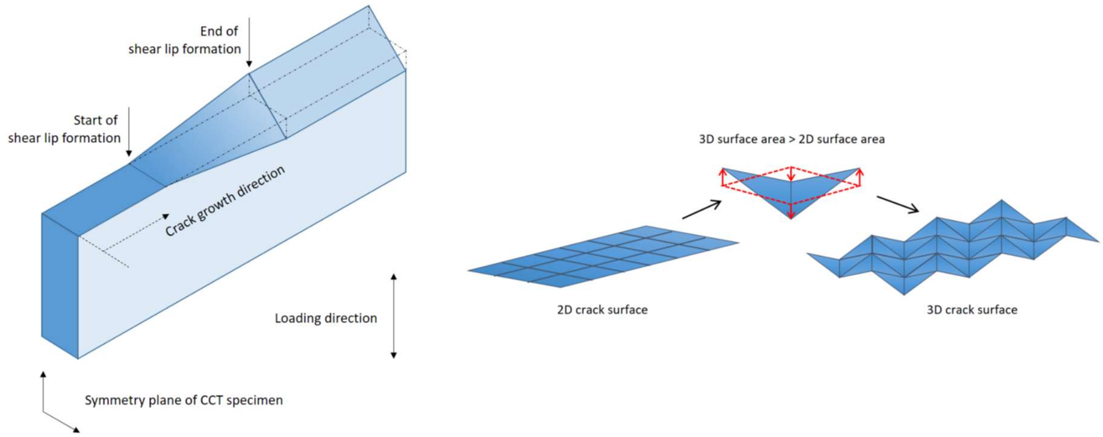

Figure 1: Illustration of the fracture surface area increase as results of shear lip formation (left) and as result of fracture surface roughness (right) [7]

With the shear lip formed along a plane of about 45° relative to the major fracture plane, this correction factor increases linearly from unity at the start of the shear lip formation to √2 for a fully slanted crack plane.

Because current literature reveals no consensus on the proper definition of roughness to calculate the exact fracture surface area, the method adopted here considers a change in roughness relative to a reference roughness, i.e. $\lambda = r_a(a)/r_a(0)$.

Plastic volume

The volume of plasticity at the crack tip develops along with the developing crack, hence it can be expressed as function of the crack length, i.e. *f(a)*. Rather than exactly modelling the plastic volume, a very common model proposed by Irwin [8] is adopted. Irwin modelled the plastic volume as a cylinder with the height of the thickness t and a radius r_p

$$r_p = \frac{1}{\alpha\pi}\left(\frac{K_{max}}{S_{yield}}\right)^2 = \frac{1}{\alpha\pi}\left(\frac{\beta S_{max}\sqrt{\pi a}}{S_{yield}}\right)^2 = \frac{\beta^2}{\alpha}a\left(\frac{S_{max}}{S_{yield}}\right)^2 \quad (7)$$

with $\alpha = 1$ for plane stress conditions and $\alpha = 2$ for plane strain conditions. The corresponding plastic volume then is

$$V_p = \pi r_p^2 t = \pi\frac{\beta^4}{\alpha^2}a^2\left(\frac{S_{max}}{S_{yield}}\right)^4 = \frac{\pi}{\alpha^2}a^2 sec^2\left(\frac{\pi a}{W}\right)\left(\frac{S_{max}}{S_{yield}}\right)^4 \quad (8)$$

Note the difference in plastic volume between plane stress and plane strain conditions: the first can be 4 times greater than the latter!

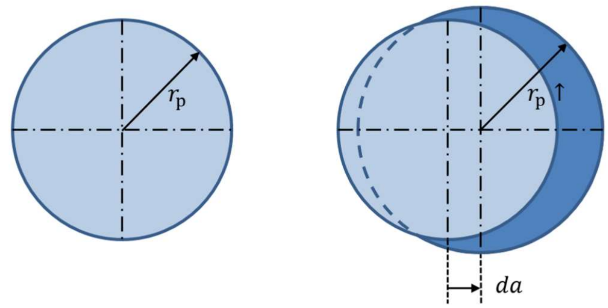

Figure 2: Schematic view of Irwin's cylindrical plastic zone [7], and the growth as a crescent volume increase caused by a combination of crack tip translation *da* and increase in radius r_p.

Mean plastic energy density

The plastic strain energy dissipation in the energy balance can be expressed per load cycle as

$$\frac{dU_p}{dN} = \frac{dU_p}{dV_p}\frac{dV_p}{dN} = \frac{dU_p}{dV_p}\frac{dV_p}{da}\frac{da}{dN} \tag{9}$$

where the dV_p/dN represents the crescent plastic volume increase illustrated in Figure *2*, and dU_p/dV_p constitutes the mean plastic energy density in that volume V_p. The change in mean plastic energy density is not constant, but changes with the development of crack length and plastic volume. Here, the higher the yield strength of a material, the higher the energy density in the volume. This was demonstrated through analysing different fatigue crack propagation experiments on centre-crack tension specimen tested under constant amplitude (CA) fatigue loading, see Table *1*.

Table 1: Fatigue test and modelling parameters of several CCT fatigue tests, indicating different CA fatigue load spectra and material/alloy choices.

Test	*Material*	*W [mm]*	*t [mm]*	a_0 *[mm]*	S_{max} *[MPa]*	*R*	S_{yield} *[MPa]*	*Ref*
1	2024-T3	160	6.1	13.18	60	0	324	[7]
2	2024-T3	160	6.1	11.25	80	0.3	324	[7]
3	7075-T6	160	3.2	6.35	48.8	0.1	503	[5,9]
4	Fe510Nb	100	10	10.0	113	0.5	460	[10]

Through solving the energy balance of equation (1) in discrete form (expressed per cycle *d/dN*) with *dUp/dVp* as the unknown, resulted for the four tests in Table *1*, in the *dUp/dVp-a* curves shown in Figure *3* (left). When scaling these curves following

$$\frac{dU_p}{dV_p}_{scaled} = \frac{dU_p}{dV_p}\left(\frac{S_{max}}{S_y}\right)^2 \tag{10}$$

yields the curves illustrated in Figure *3* (right). Although the curves do not entirely overlap, there is a consistent trend visible that seem to illustrate that the average energy density in the plastic zone reduces to zero at about *2a/W*~0.65. This most likely is attributed to the crack length where the plastic volume becomes infinite, often referred to as net section yielding. The elastic analysis underneath the energy balance, and the assumption of small scale yielding, underneath Irwin's approximations essentially no longer hold.

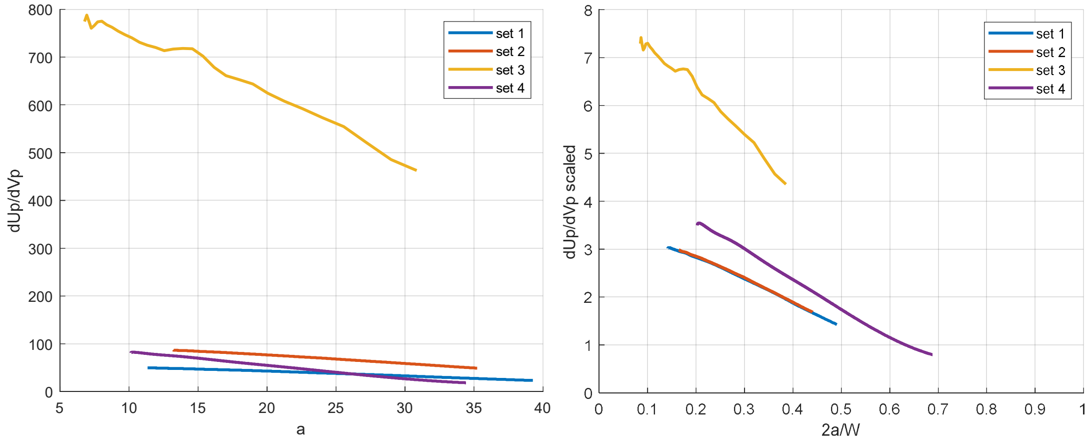

Figure 3: dU_p/dV_p curves determined from the tests in Table 1 plotted against crack length (left) and dU_p/dV_p scaled with $(S_{max}/S_y)^2$ plotted against crack length normalized by width (right).

Change in elastic energy stored

In a perfect elastic case where no energy is dissipated through plasticity or fracture surface formation, all work applied to the specimen, is stored as elastic energy, which upon unloading would be returned by the specimen as work applied to the test machine. In this case, $dU/dN = 0$, because no energy is permanently dissipated at the end of the load cycle. However, when energy is dissipated, particularly in plasticity, unloading will not return all elastic energy stored in the panel. The plastic volume will not return to its original volume, prohibiting all elastic energy to be released. This is generally observed as development of residual stresses due to plastic deformation, which in the physics concept of equation (1) means $dU_e/dN \neq 0$.

This loss in elastic energy develops proportionally to the development of plastic volume, and because the plastic strains are much larger than the elastic strains, the plastic energy dissipation is much greater than the elastic energy loss. For the current discussion, focussing on constant amplitude loading, we may assume that the elastic energy loss dU_e/dN is linearly proportional to the plastic energy dissipation dU_p/dU. As a consequence, the elastic energy loss term can be neglected in the energy balance, when included in the plastic energy dissipation term through a constant scale factor.

Keep in mind, however, that this certainly will not hold for variable amplitude loading, where distinct peak loads affect the elastic energy stored, equivalent to the residual stresses, over successive cycles. In that case, a cycle-by-cycle analysis may be required to evaluate the term dU_e/dN.

Physic of energy balance versus engineering prediction

The continuous energy balance of equation (1) can be discretised through frequency to a balance per load cycle, combining equations (6) and (9), i.e.

$$\frac{dU}{dN} = \frac{dU}{da}\frac{da}{dN} = \frac{dU_p}{dV_p}\frac{dV_p}{da}\frac{da}{dN} + \frac{dU_a}{dA}t\gamma\lambda\frac{da}{dN} \tag{11}$$

This does, however, not change the continuous nature of the energy balance as formulated in equation (1), illustrated that in equation (11) the term da/dN appears in all components of the equation and therefore can be cancelled out. Hence, fatigue cycles as integer variable drop out of the energy balance, which means that the energy balance in itself does not constitute a predictive capability for the crack growth rate da/dN!

To understand this observation, take the analogy of a falling marble [7], where the potential energy described by $m \cdot g \cdot h$, is in balance with kinetic energy described by $½ m \cdot v^2$. With the decrease in altitude, the potential energy reduces, while the kinetic energy through the velocity increases. The total energy remains constant.

If we consider now a staircase, from which this marble drops, step by step, then the apparent discrete nature of the marble's motion, is not changing the energy balance governing the process of falling. At each step the marble's drop is described by the same balance between both energy components.

This analogy illustrates the difference between a theory from physics, describing a process in nature, and an engineering prediction. The engineer aims to predict how often a process may be repeated, as a tangible measure in eg. aircraft operation, while the physical theory only describes the mechanisms governing the process itself. Hence, the energy balance in equation (1) or (10) does not allow prediction of *da/dN* in itself.
Another interpretation of this observation is that the energy balance describes the resistance of the material, specimen or structure to fatigue crack growth. Without an external driving quantity (in literature often referred to 'driving force', which is not the proper measure in physics) to which this resistance can be related, the consequence (measured in crack growth) cannot be predicted.

Analogy of a sliding box

Consider a box sliding over the floor due to an applied force, illustrated in Figure *4*. In physics, this problem can be described by the equations of motion. In the initial condition with the box at rest, the force applied must overcome stiction defined by static friction coefficient, which is analogous to the threshold in fatigue crack growth: a driving force below such threshold will not move the box or increment a crack.

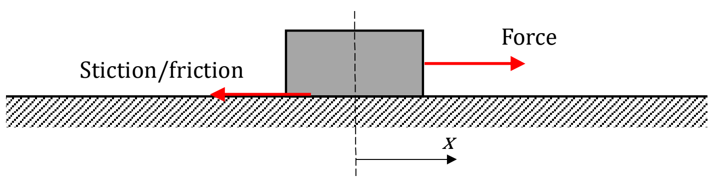

Figure 4: Illustration of the stiction/friction analogy with a sliding box.

Once the static friction is overcome by the applied force, the box will accelerate as described by Newtonian mechanics

$$z(t) = \frac{F_{net}(t)}{M} = \frac{F(t) - F_f}{M} \tag{12}$$

Note that the kinematic friction in this case may be different from the static friction; often the friction force while the box is in motion is lower than the initial static friction force that the force has to overcome. Integration of the acceleration yields the velocity $v(t) = v_0 + z(t) \cdot t$, while integration further will yields the position $x(t)=x_0 + v(t) \cdot t + z(t) \cdot t^2$. For an arbitrary case of mass with force, the results are illustrated in Figure *5*.

In this analogy, the load cycle in the sliding box concept, is related to the fatigue load cycle, or more specifically the cyclic work applied in the fatigue crack growth scenario. Because the fatigue load cycle is often considered to be a sinusoidal shape, from equation (3) it then follows that the corresponding cycle in applied work is proportional to $sin^2(t)$.
The stiction represents the threshold, i.e. the minimum value required before the box will slide or the crack will increment, while the friction represent the energy dissipation associated with the fracture surface formation together with all other dissipating mechanisms, of which plasticity is the dominant one. The distance travelled by the sliding box in the analogy represents the crack increment, i.e. $x \sim da/dN \sim \Delta a/\Delta N \sim \Delta a$.

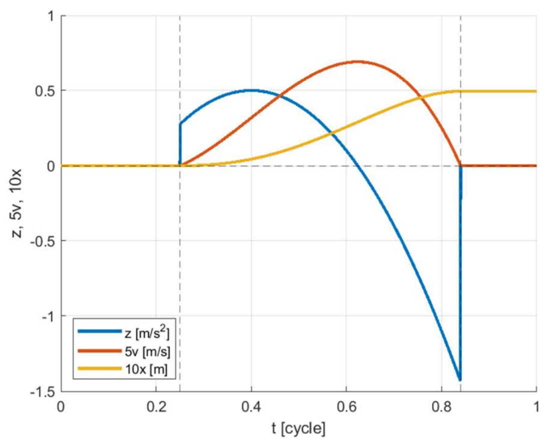

Figure 5: Arbitrary case of sliding box pulled by sinusoidal force, where stiction (threshold) is exceeded at t=0.25s, acceleration of the box is governed by friction (<stiction), until it comes at rest at t=0.84s until the end of the load cycle [7].

PREDICTION OF CRACK GROWTH

To predict crack growth using the above concepts from physics, i.e. the energy balance and the sliding box analogy, crack growth theoretically can be predicted in two ways: (i) the crack growth can be predicted using the sliding box analogy based on assumed levels of stiction and friction, or (ii) through an iterative scheme between the sliding box analogy to quantify the 'driving force' (elastic work) and the energy balance to describe the corresponding energy dissipation, i.e. the friction.

In the first case, the stiction coefficient C_{stic} is taken close to unity, such that the available strain energy for initiating the crack increment process is $C_{stic}U_{o,max}$. Essentially, this forms an initial trigger to increment the crack by little extent. The larger the volume of the cracked plate relative to the crack tip region, the closer the stiction coefficient should be taken to unity. The friction can be defined equivalent to known physical crack closure levels such that $C_{fric} = S_{op,phys}/S_{max}$ [13], hereby including the effect of S_{max} and the effect of stress ratio R. The net force in the analogy of the sliding box then is defined by

$$F_{net}(t) = (1 - C_{fric})U_{a,max} \tag{13}$$

while the mass M in the analogy and equation (12) is defined by the volume of the specimen, i.e. $M = WLt$. Through the equations of motion, the travelled distance of the sliding box is assumed to be the crack increment, i.e. $x = da/dN = \Delta a/\Delta N = \Delta a$. With these da/dN results, the crack growth curve can be derived through quantifying a and N until the final crack length, while the corresponding ΔK_{eff} can be calculated for the computed a versus N, complementing the ranged dU/dN and U_{max}. The methodology following both options is conceptually illustrated in the flow diagram in Figure 6.

DEMONSTRATING PREDICTIVE CAPABILITY

The crack growth rate da/dN calculated through either of the two procedures can be integrated to a versus N curves, which are compared to the experimental crack growth curves in Figure 7, while with the corresponding S_{max} and S_{op}/S_{max} relation one can determine the ΔK_{eff} , which is illustrated in Figure 8.

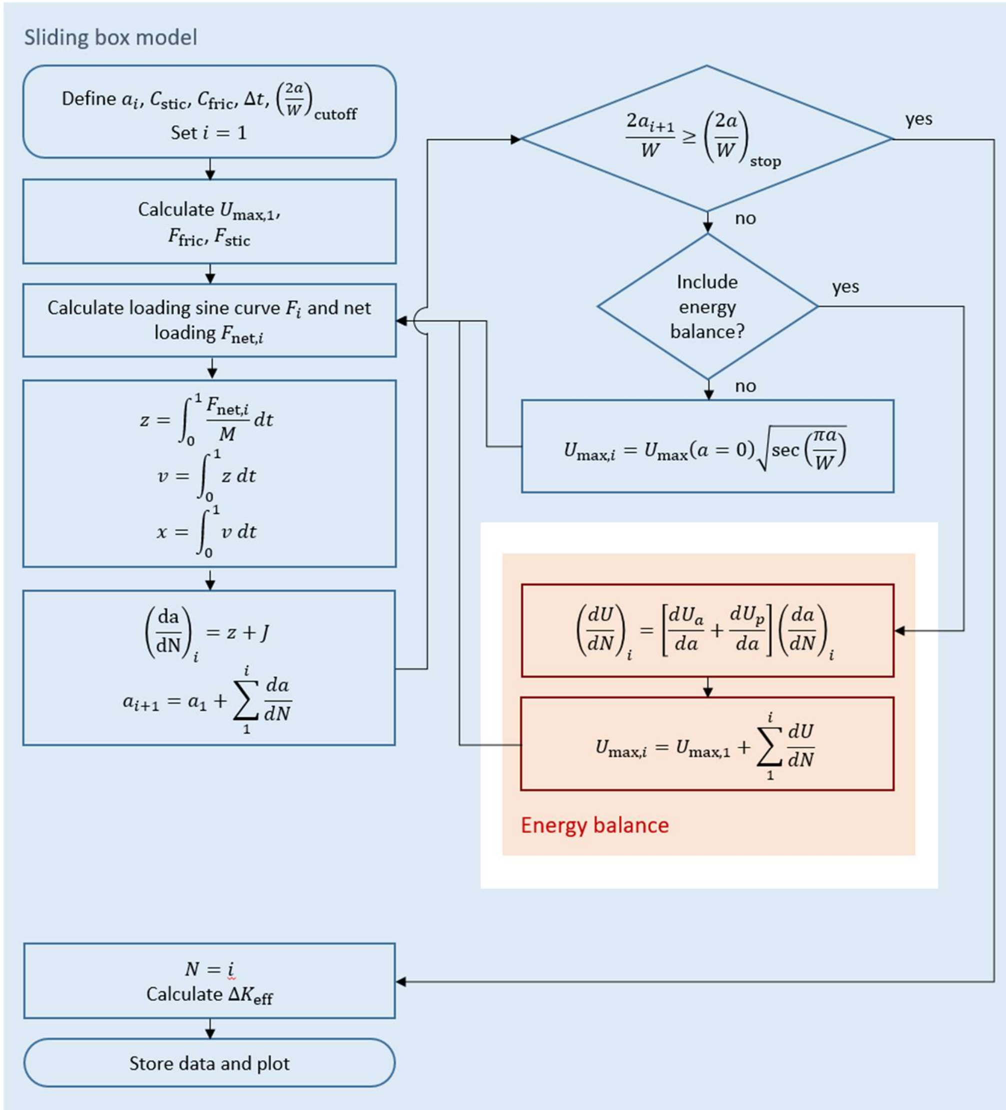

Figure 6: Flow chart illustrating the sliding box analogy for predicting *da/dN,* with and without utilizing the energy balance for estimating the energy dissipation [7].

CONCLUSIONS

This study has demonstrated that in essence through describing the crack growth and associated energy dissipating mechanisms through proper concepts from physics, crack growth predictions are possible with only mechanical properties of the metal, without the use of a Paris crack growth resistance curve. Once the crack growth is predicted, the associated effective stress intensity factor range can be calculated, to develop the corresponding Paris curves after the fact.

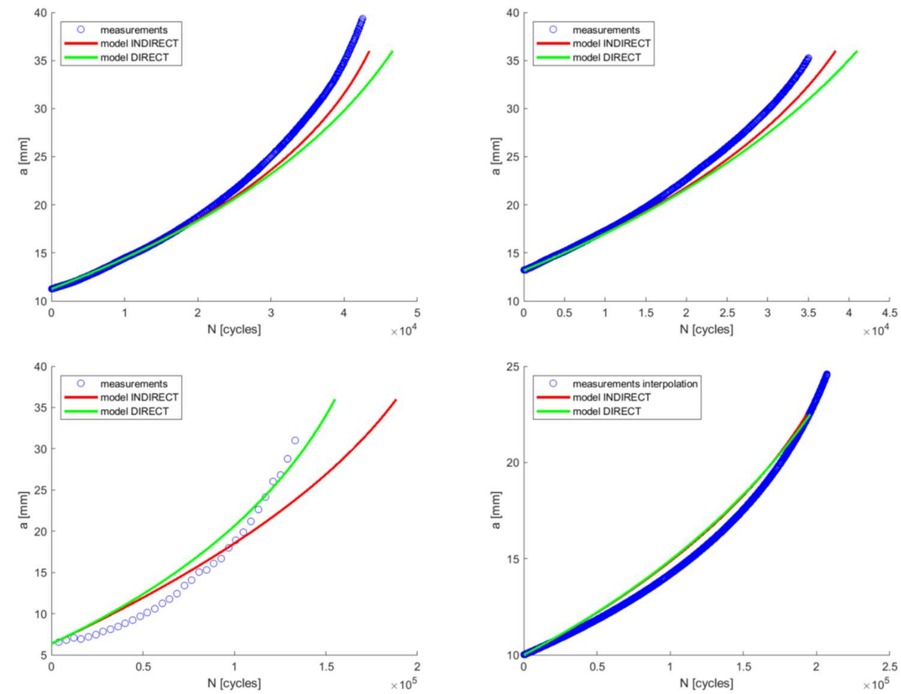

Figure 7: Crack growth curves predicted by Van Kuijk through both the direct and indirect method, in comparison with data from fatigue crack growth experiments [7].

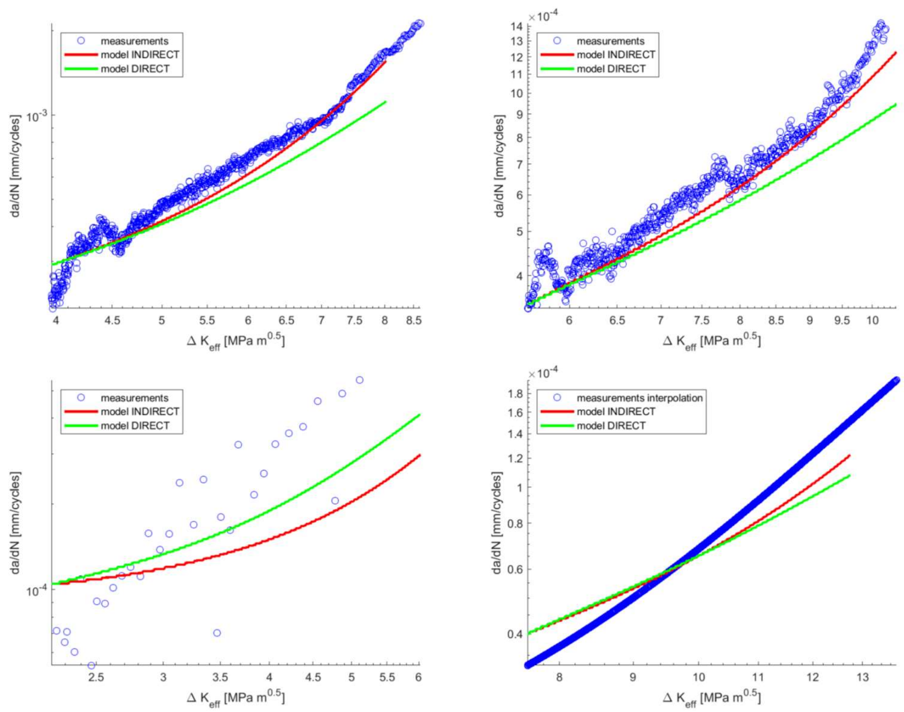

Figure 8: Paris resistance curves predicted by Van Kuijk through both the direct and indirect method, in comparison with data from fatigue crack growth experiments [7].

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