FATIGUE LIFE PREDICTION OF METALLIC MATERIALS USING THE TANAKA-MURA-WU (TMW) MODEL

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Research Background

- Traditional fatigue design is experiment-based
- Drawbacks
- Fatigue testing data scattering
- A large number of coupons required
- Long development cycle of new materials
- Computational fatigue design has been proposed and received an increasing interest
- Physics-based models are more effective because they are almost independent of fatigue testing

Research Objectives

- **Tanaka and Mura (TM) Model**
- Fatigue damage in essence is associated with plastic deformation as a result of dislocation interactions
- Fatigue crack nucleation is caused by the accumulation of irreversible dislocation dipoles within the slip band
- Drawback: The model evaluates the plastic strain by integration of displacement, which results in a physical dimension of m^2 in the strain unit

- **Tanaka-Mura-Wu (TMW) Model**
- It modifies the true plastic strain expression by taking the slip distance evaluated as the number of dislocations multiplied by the Burger's vector, then the plastic strain is the slip distance divided by the distance which eventually is the crack length
- It derives the fatigue crack nucleation life based on the same dislocation pileup mechanism, but which depends on the Burger's vector, elastic modulus, and surface energy
- Application to the full range of fatigue life, from low cycle fatigue (LCF) to high cycle fatigue (HCF) for different materials

Fatigue Testing Data Fitting

- Basquin Equation
- A power-law relationship between the stress amplitude and fatigue life

$$\frac{\Delta\varepsilon_e}{2} = \frac{\Delta\sigma}{2E} = \frac{\sigma_f{}'}{E}(2N_f)^b$$

where $\sigma_f{}'$ is fatigue strength coefficient and b is fatigue strength exponent.

- Coffin-Manson Equation
- A power-law relationship between the plastic strain amplitude and fatigue life

$$\frac{\Delta\varepsilon_p}{2} = \varepsilon_f{}'(2N_f)^c$$

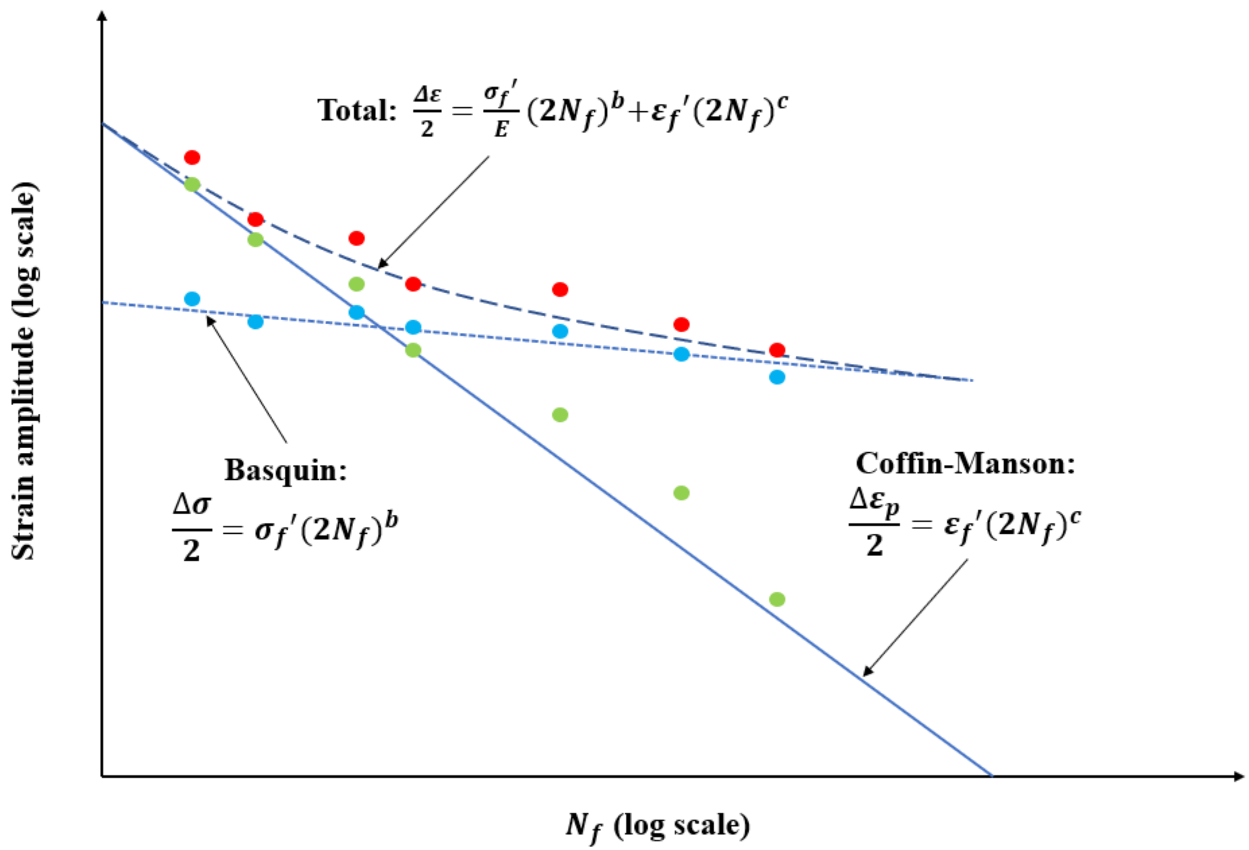
where $\varepsilon_f{}'$ is fatigue ductility coefficient and c is fatigue ductility exponent.

Total Strain – Life Relation

- Coffin-Manson-Basquin Equation

$$\frac{\Delta\varepsilon}{2} = \frac{\sigma_f{}'}{E}(2N_f)^b + \varepsilon_f{}'(2N_f)^c$$

- It is the combination of Basquin Equation for high cycle fatigue (HCF) and Coffin-Manson Equation for low cycle fatigue (LCF) ($\frac{\Delta\varepsilon_p}{2} > 10^{-3}$)
- It represents the full-range fatigue strain – life behavior



Total strain-life curve composed of Coffin-Manson and Basquin relations, each represents a linear regression of experimental data

Chemical compositions of studied materials

Material	Element (wt.%)				
Inconel 617	Ni	Cr	Co	Mo	Fe, Mn, Si, etc.
	≥44.5	20-24	10-15	8-10	Bal.
SAE 1020	Fe	Mn	C	S	P
	99.08-99.53	0.3-0.6	0.17-0.23	≤0.05	≤0.04
SAE 4340	Fe	Ni	Mn	Cr	Mo, C, Si, etc.
	96	1.83	0.7	0.7-0.9	Bal.

Fatigue Specimens

- Inconel 617 — A solid-solution alloy, hot rolled, followed by annealing
- SAE 1020 — A low-carbon steel, hardened by heating between 815°C and 871°C, followed by water quenching, and then tempered
- SAE 4340 — An alloy steel, heated at 830°C for half hour in salt, followed by water quenching and then tempered at 430°C in salt with air cooling

Fatigue properties of studied materials

Material	σ_f' (MPa)	b	ε_f'	c
Inconel 617	1519.55	-0.104	0.50	-0.57
SAE 1020	815	-0.110	0.25	-0.54
SAE 4340	1898	-0.090	0.67	-0.64

Tanaka-Mura (TM) Model

According to the energy criteria, the entire pileup bursts into a crack once the stored energy in the material volume becomes equal to the energy to form new crack surfaces

$$N_c = \frac{4\mu w_s}{\pi(1-\nu)a}(\Delta\tau - 2k)^{-2}$$

$$N_c = \frac{4\pi(1-\nu)w_s a^3}{\mu}\Delta\gamma^{-2}$$

where w_s is the specific surface energy for unit area, k is lattice-friction stress and a is the half grain size

Tanaka-Mura Model

Main drawback

- Plastic strain is obtained by integration of the displacement so that it has a physical dimension of m^2

$$\Delta\gamma = \frac{(1-\nu)(\Delta\tau - 2k)a^2\pi}{\mu}$$

- Therefore, this model is not useable in practice since no strain is measured in unit of m^2

Tanaka-Mura-Wu (TMW) Model

- The dislocation pileup strain formulation has been revised, which solves the unit problem
- By definition, strain is the displacement over the distance
- The slip distance is evaluated as the number of dislocations times the Burger's vector ***b***
- Plastic strain is the slip distance divided by the distance which eventually is the crack length

$$\Delta\gamma = \frac{2(1-\nu)(\Delta\tau - 2k)}{\mu}$$

Tanaka-Mura-Wu (TMW) Model

- According to the Griffith energy criterion, once the stored energy in the material volume becomes equal to the energy to form new crack surfaces, the entire pileup bursts into a crack, then the fatigue life can be expressed as

$$N_c = \frac{8(1-\nu)w_s}{\mu b} \Delta\gamma^{-2}$$

$$N_c = \frac{2\mu w_s}{(1-\nu)b} (\Delta\tau - 2k)^{-2}$$

Tanaka-Mura-Wu (TMW) Model

- In engineering practice, fatigue life is mostly assessed through uniaxial fatigue testing of material coupons with certain surface roughness
- In the uniaxial test, the plastic shear strain has a relationship with the normal plastic strain by the Taylor's factor: $\gamma = \sqrt{3}\varepsilon_p$ and $\sigma = \sqrt{3}\tau$ based on the plastic energy is equal for both shear and normal deformation, $\tau\gamma = \sigma\varepsilon_p$
- A surface roughness factor R_s needs to be introduced in the model

Tanaka-Mura-Wu (TMW) Model

$$N_c = \frac{8(1-\nu)R_s w_s}{3\mu \boldsymbol{b}} \Delta\varepsilon_p{}^{-2}$$

$$N_c = \frac{6\mu w_s}{(1-\nu)\boldsymbol{b}} (\Delta\sigma - 2\sigma_0)^{-2}$$

where $\sigma_0 = \sqrt{3}k$, k is lattice-friction stress, but in practice σ_0 is calibrated to the stress level at 10^7 cycles

The machined surface roughness has an effect of $R_s \approx 1/3$ and the electropolished surface $R_s = 1$

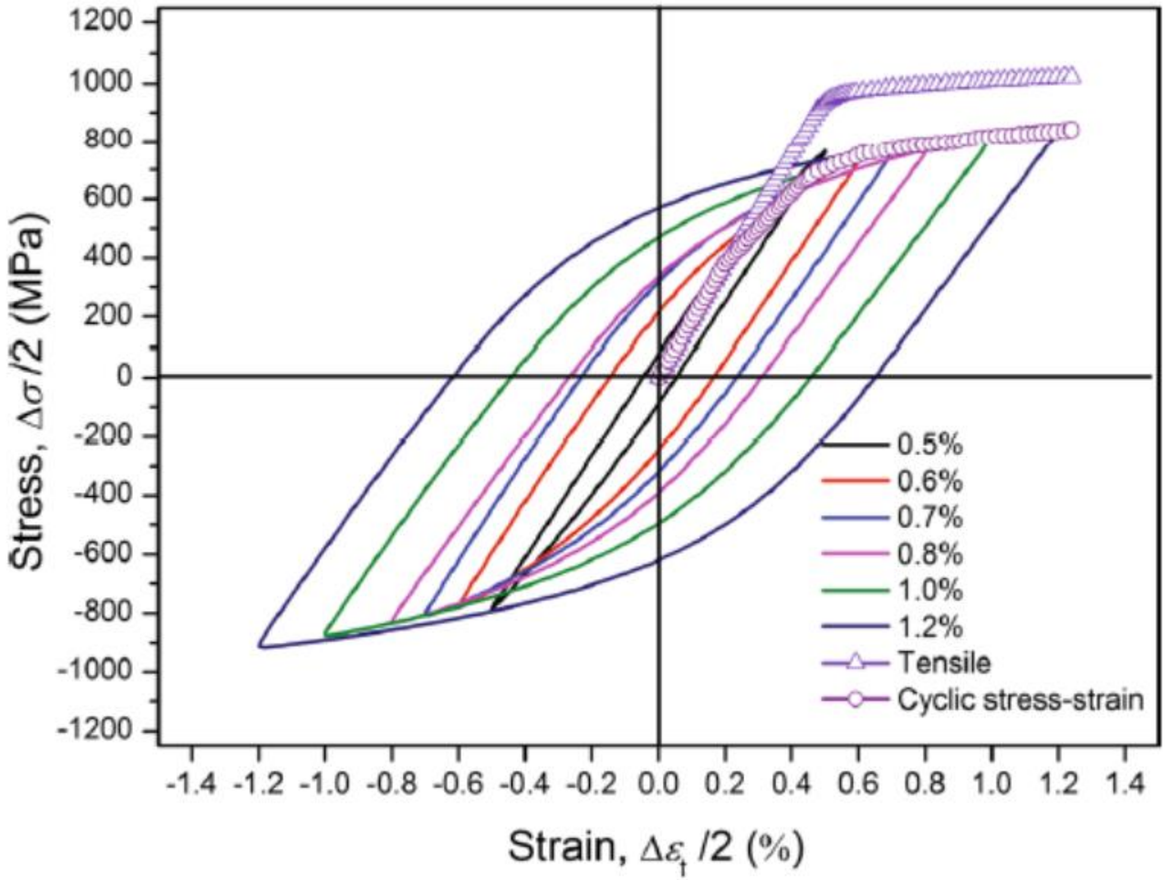
Ramberg-Osgood Equation

During fatigue, the cyclic stress - strain curve (CSSC) can be expressed by the Ramberg-Osgood equation

$$\frac{\Delta\varepsilon}{2} = \frac{\Delta\sigma}{2E} + \frac{\Delta\varepsilon_p}{2} = \frac{\Delta\sigma}{2E} + \left(\frac{\Delta\sigma}{2K'}\right)^{1/n'}$$

where K' is plastic strength coefficient and n' is hardening exponent

Ramberg-Osgood equation is established by connecting the peak points of the stabilized hysteresis loop (usually at the half-life point) without fracture



Cyclic stress-strain curve from half-life hysteresis loops

Cyclic hardening coefficients of studied materials

Material	K' (MPa)	n'
Inconel 617	1910	0.12
SAE 1020	941	0.18
SAE 4340	1950	0.13

Determination of Model Parameters

In the TMW model, The surface energy w_s is determined by

$$w_s = w_s(T_m) + \phi(T)\frac{RT_m}{A}$$

where $w_s(T_m)$ is the surface energy at the melting temperature, A is the surface area per mole of surface atoms, $\phi(T)$ is a temperature-dependent parameter having a value ranging from 0 (at the melting point) to 1 (at the absolute zero-degree Kevin), and R is the universal gas constant

For simplicity and consistency, in this study, it is assumed that $\phi(T)$ = 0.85 at room temperature (20°C)

The TMW model parameters of studied materials

Material	ν	μ (GPa)	w_s (J/m^2)	σ_0 (MPa)	R_s	$\mathbf{b}$ (10^{-10} m)
Inconel 617	0.34	82.46	2.335	275	1/3	2.48
SAE 1020	0.29	79.45	2.373	100	1/3	2.48
SAE 4340	0.3	76.92	2.373	462	1/3	2.48

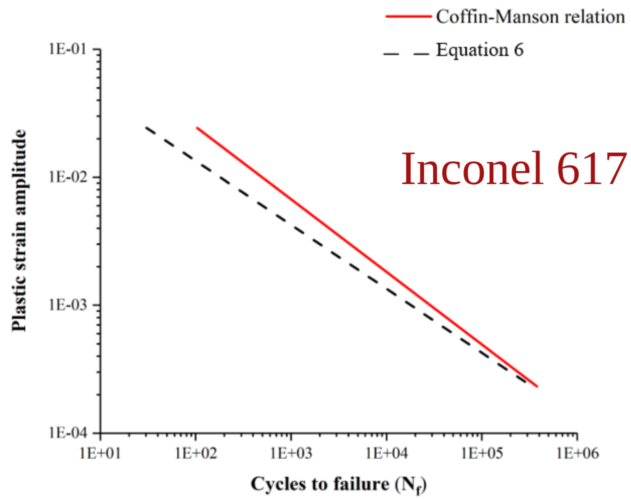
LCF Life Prediction

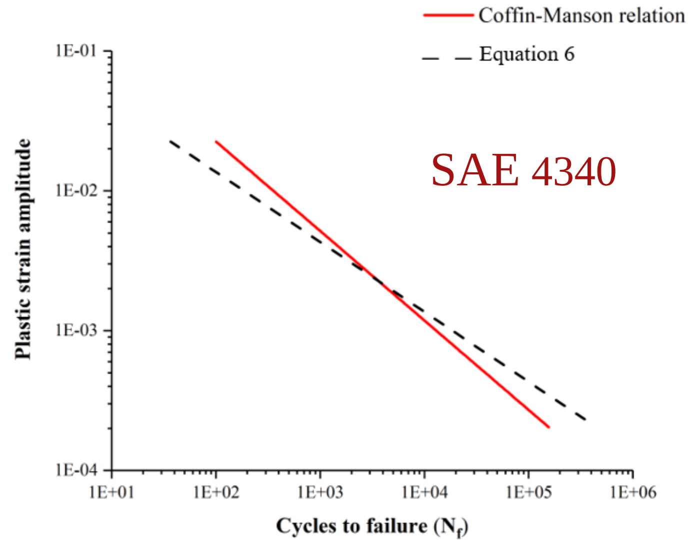
- Comparison between the MTW model (for strain) and the Coffin-Manson equation

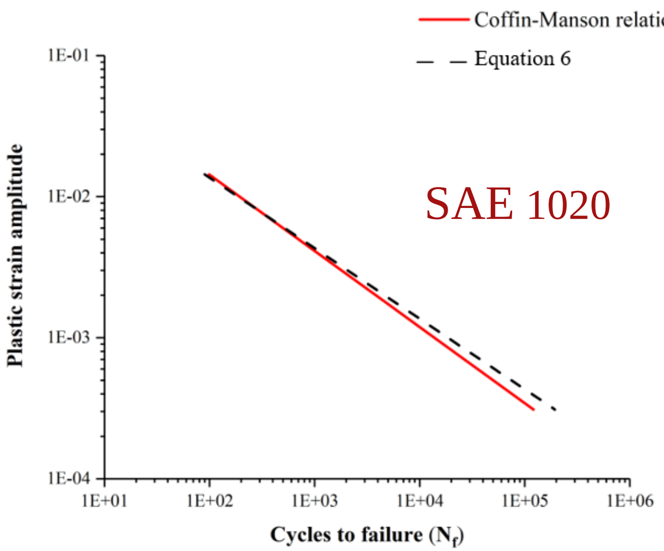
$$N_c = \frac{8(1-\nu)R_s w_s}{3\mu b} {\Delta\varepsilon_p}^{-2}$$

$$\frac{\Delta\varepsilon_p}{2} = {\varepsilon_f}'(2N_f)^c$$

- Since the calculation of LCF life is independent of fatigue testing, the prediction of the TMW model is deemed to be a class-A prediction







Comparison of fatigue life prediction from the TMW model (strain) with the Coffin-Manson equation

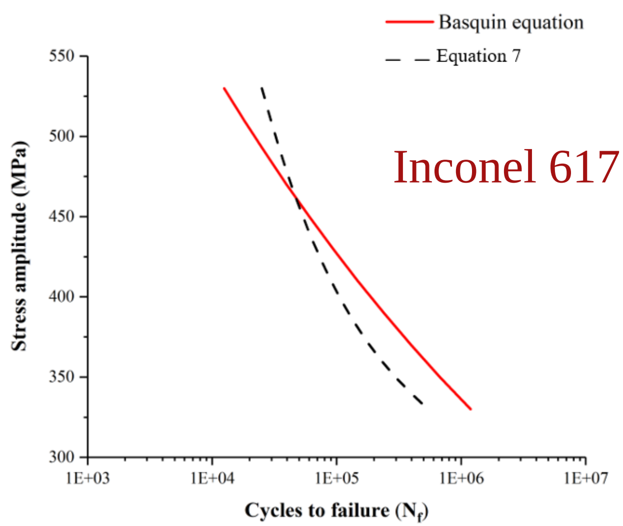
HCF Life Prediction

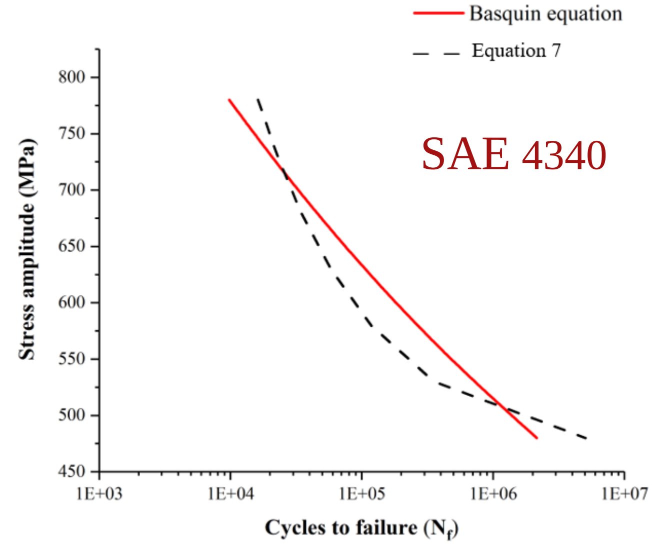
- Comparison between the MTW model (for stress) and the Basquin equation

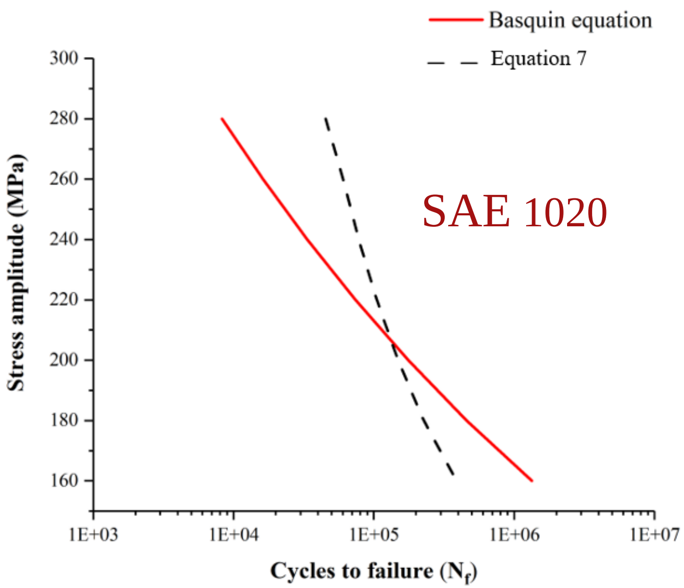
$$N_c = \frac{6\mu w_s}{(1-\nu)b}(\Delta\sigma - 2\sigma_0)^{-2}$$

$$\frac{\Delta\sigma}{2} = \sigma_f{}'(2N_f)^b$$

- The microstructural slip resistance parameter σ_0 has to be calibrated to a known test result, corresponding to the stress magnitude at the cycle of 10^7.







Comparison of fatigue life prediction from the TMW model (stress) with the Basquin equation

- In the LCF range, the agreement between the prediction and the experimental behavior (the best-fit Coffin-Manson correlation) is within the typical fatigue scatter factor of 2
- The theoretical prediction of the TMW model agrees well with the Basquin curve in the HCF region ($N_f > 10^4$ cycles) with a typical scatter factor of 2 for the studied materials, except for SAE 1020
- The Basquin equation does not have a true fatigue endurance limit, therefore, the extrapolation of the Basquin equation fitted with the data at intermediate stress levels may be erroneous towards the low stress region

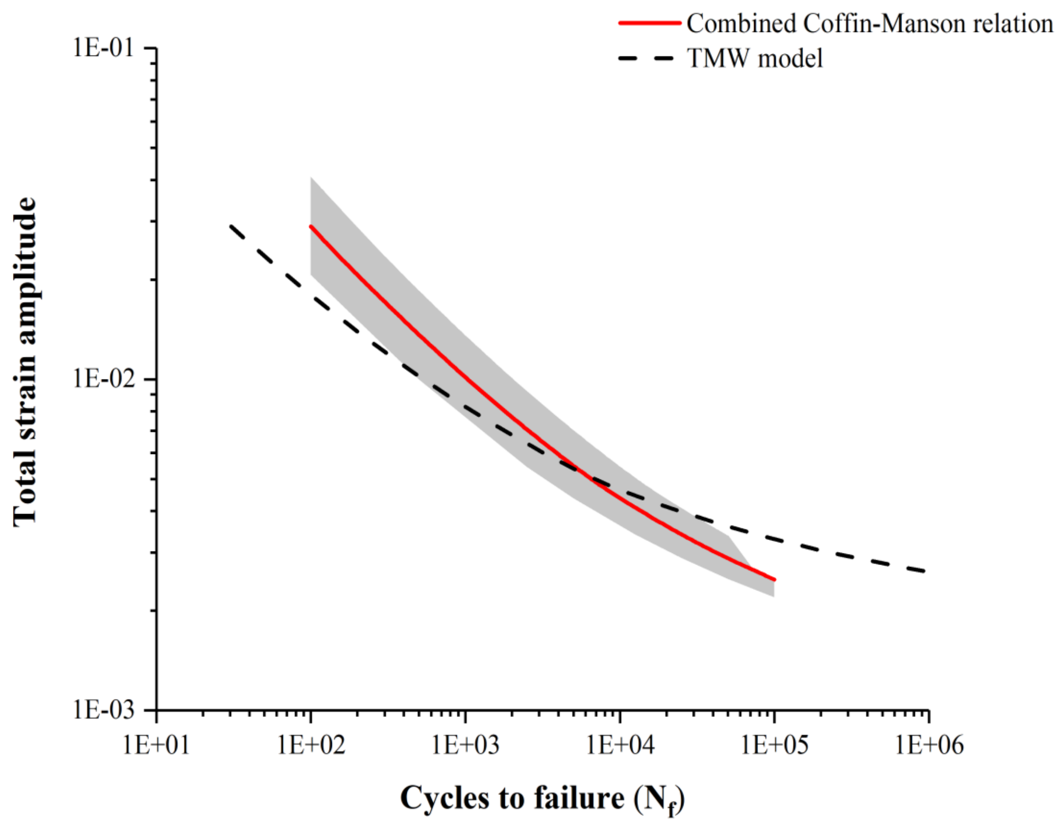
Total Life Prediction

- Comparison between the MTW model and the Coffin-Manson-Basquin equation

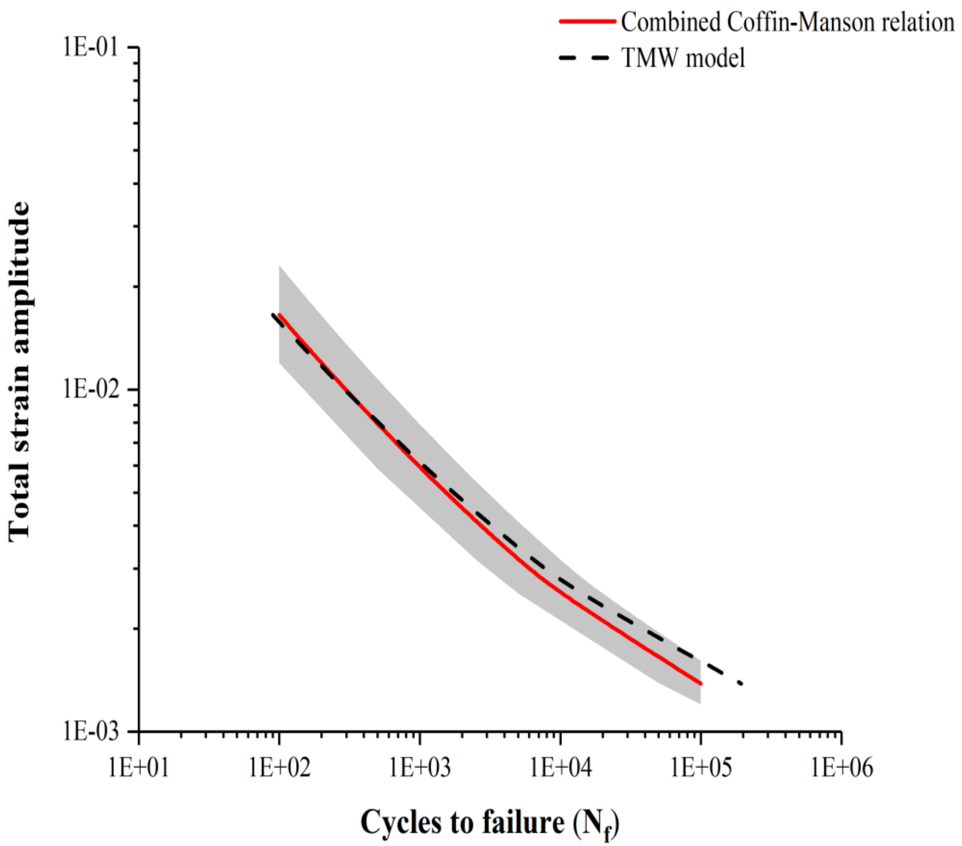
$$\frac{\Delta\varepsilon}{2} = \frac{\Delta\sigma}{2E} + \frac{\Delta\varepsilon_p}{2} = \frac{\Delta\sigma}{2E} + \left(\frac{\Delta\sigma}{2K'}\right)^{1/n'}$$

$$\frac{\Delta\varepsilon}{2} = \frac{\Delta\sigma}{2E} + \frac{\Delta\varepsilon_p}{2} = \frac{\sigma_f{}'}{E}(2N_f)^b + \varepsilon_f{}'(2N_f)^c$$

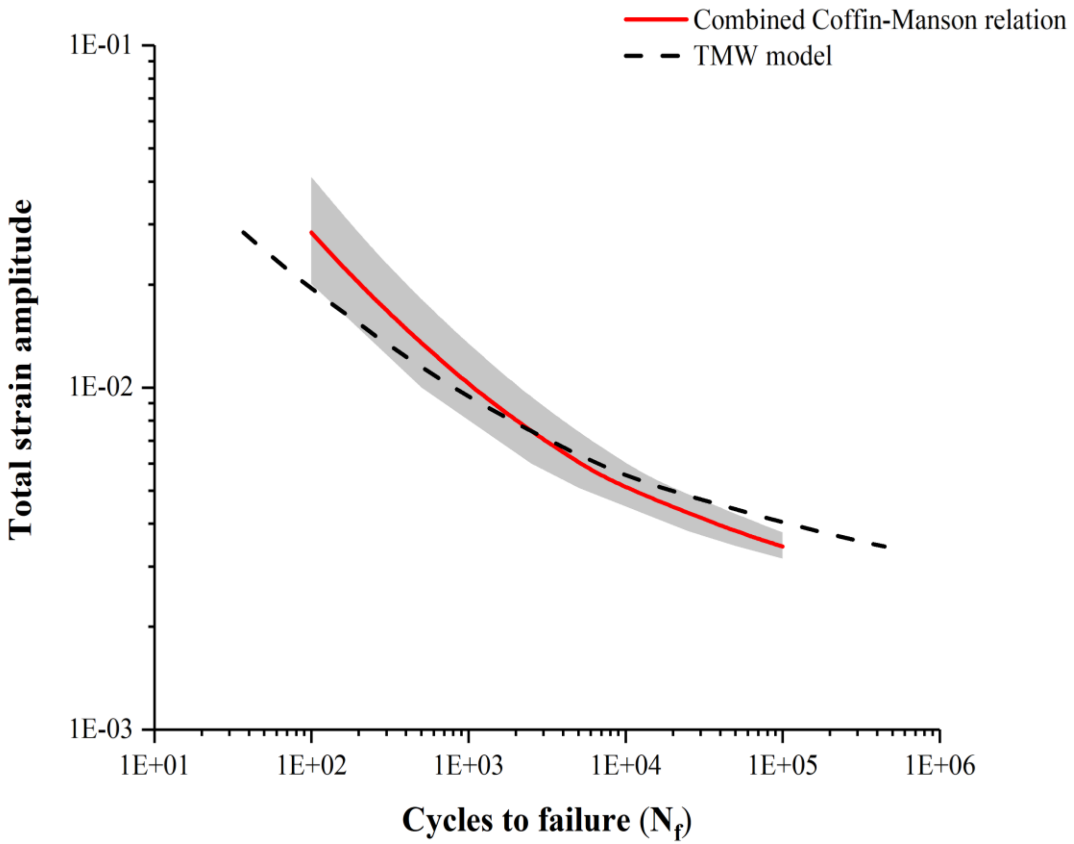
- The predictions mostly fall in the scatter band, in particular, when the fatigue life $N_f < 10^4$



Comparison of fatigue life prediction from the TMW model with Basquin and Coffin-Manson equations for Inconel 617



Comparison of fatigue life prediction from the TMW model with Basquin and Coffin-Manson equations for SAE 1020



Comparison of fatigue life prediction from the TMW model with Basquin and Coffin-Manson equations for SAE 4340

Conclusions

- The TMW model is derived from the mechanism of dislocation dipole pileup, which depends on material physical properties such as elastic modulus, Poisson's ratio, surface energy and the Burger's vector
- The plastic strain-based equation can provide class-A predictions for the LCF life, but the stress-based equation is more suitable for HCF life prediction where the lattice resistance σ_0 needs to be calibrated to one S-N curve point close to the fatigue endurance limit
- The TMW is shown to be applicable to nickel-based alloy, low-carbon steel and alloy steel within a scatter factor of 2 as compared with the Coffin-Manson-Basquin relation