ON THE DEVELOPMENT OF STATISTICAL KNOCK-DOWN FACTORS FOR WFD ASSESSMENT

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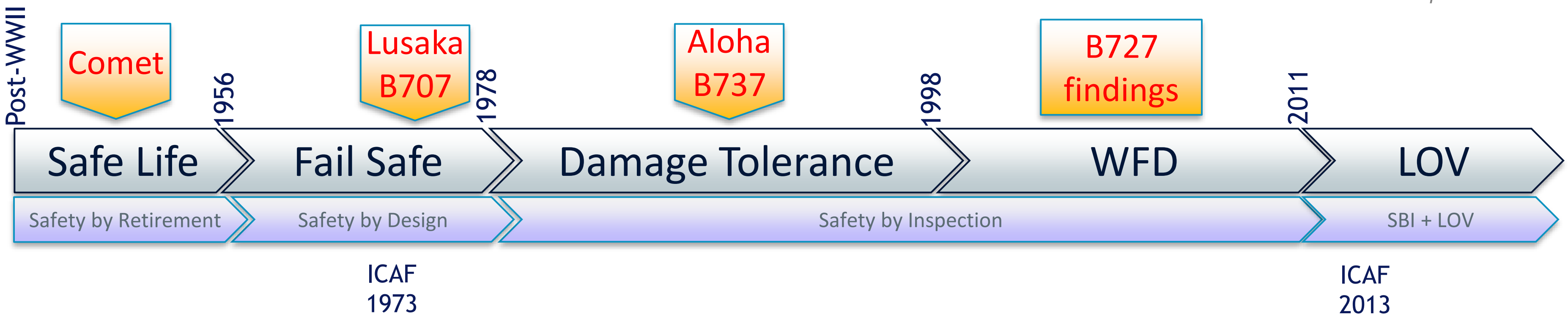
AGENDA

- INTRODUCTION
- THE KNOCK-DOWN FACTORS
- ON THE STATISTICAL NATURE OF METAL FATIGUE
- THE WEIBULL DISTRIBUTION
- DEVELOPING THE KNOCK-DOWN FACTORS STATISTICALLY
- EXAMPLES AND DISCUSSION
- CONCLUSIONS AND COMMENTS

INTRODUCTION

NOTE: This timeline depicts schematically the evolution of FAA Requirements



"It is considered that there is likely to be a finite life for fail-safe structures beyond which safety is impaired by the **onset of many interacting cracks,** a case which is not normally tested in current practice."

Maxwell, R.D.J.

"Another factor which current certification requirements do not cater for is the possibility of a **number of defects developing concurrently in adjacent structure** with potentially serious consequences for the failsafe performance of the component."

O'Brien, K.R.A. et al

WFD based on fatigue life estimation and the application of a **set of knock-down factors**.

Safarian, P.

THE KNOCK-DOWN FACTORS

SAFARIAN, P., ICAF 2013

A total of four knock-down factors were presented, but without providing details about how they were obtained

Reliability Factor

Material	95% Reliability	99% Reliability
Aluminium	0.48	0.32
Titanium and Steel (<200 Ksi)	0.37	0.22
Steel (≥ 200 Ksi)	0.26	0.12

Scale Factor

Percent of structure represented in test	Aluminium	Titanium & Steel < 200 Ksi	Steel ≥ 200 Ksi
≥ 100	1	1	1
50-100	0.84	0.79	0.73
30-50	0.76	0.69	0.61
20-30	0.71	0.63	0.53
10-20	0.6	0.5	0.4
5-10	0.5	0.4	0.3
2-5	0.38	0.28	0.18
1-2	0.33	0.23	0.14
Less than 1%	0.28	0.19	0.1

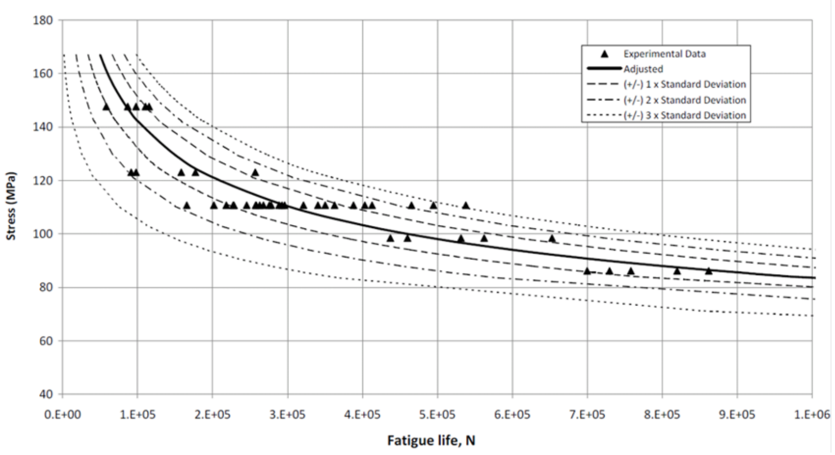
Confidence Factor = 0.7

Testing Factor = 0.7

ON THE STATISTICAL NATURE OF METAL FATIGUE

"The fatigue lives of similar specimens or structures under the same fatigue load can be significantly different"

Schijve, J.



FAA AC 23-13A states that the value for the **standard deviation** of the fatigue test life is usually taken from **historical data**. This is due to the **high cost of testing sufficient numbers** of full-scale test articles to develop an accurate value for the standard deviation.

In late 1960es and early 1970es, **Whittaker et al** compiled the then available results of fatigue tests and, after a statistical treatment, concluded that for most structural applications, reasonable values of the **Weibull shape parameter** are:

α = **4.0** for **aluminum** structures

α = **3.0** for **titanium** and **low strength steel**

α = **2.2** for **high strength steel**

THE WEIBULL DISTRIBUTION

A Statistical Distribution Function of
Wide Applicability

By WALODDI WEIBULL,[1] STOCKHOLM, SWEDEN

"Weibull probability data analysis is probably the **most widely utilized technique** of processing and interpreting life data. One of many advantages is the **flexibility** of the Weibull distribution, **easy interpretation of the distribution parameters**, and their relation to the failure rates and the bathtub curve concepts."

O'Connor et al

Probability Density Function (PDF)

$$f(x) = \begin{cases} \frac{\alpha}{\beta^{\alpha}} x^{\alpha-1} e^{-(x/\beta)^{\alpha}} & x \geq 0 \\ 0 \quad \cdots\cdots & x < 0 \end{cases}$$

α = shape parameter
β = scale parameter

Cumulative Density Function (CDF)

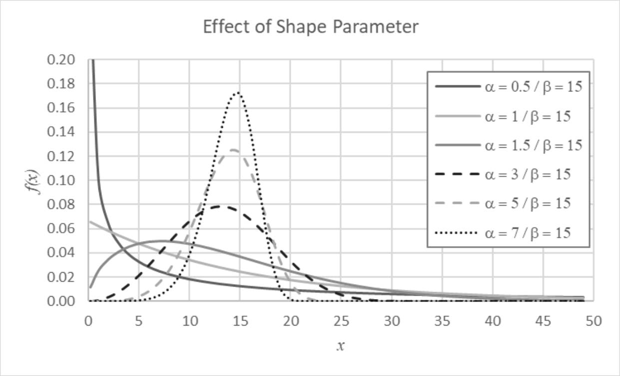
$$F(x) = 1 - e^{-(x/\beta)^{\alpha}}$$

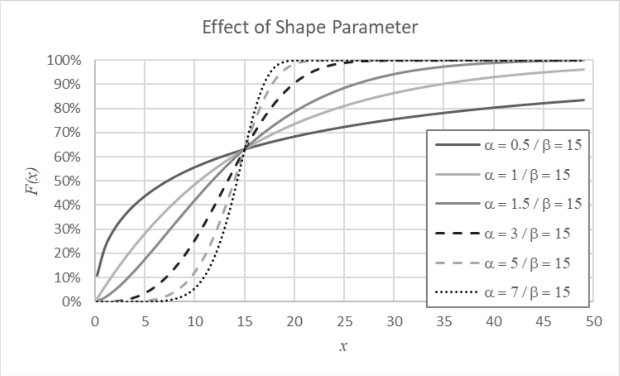
Reliability

$$R(x) = e^{-(x/\beta)^{\alpha}}$$

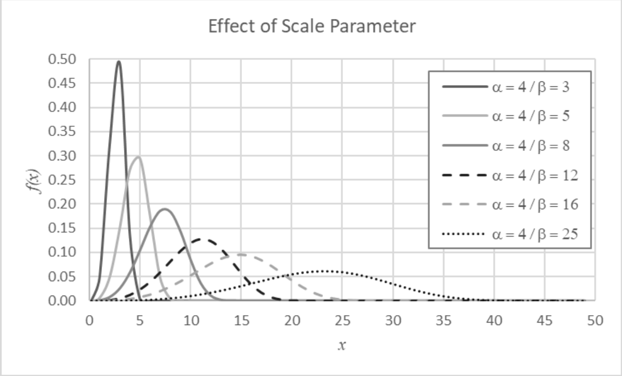
THE WEIBULL DISTRIBUTION

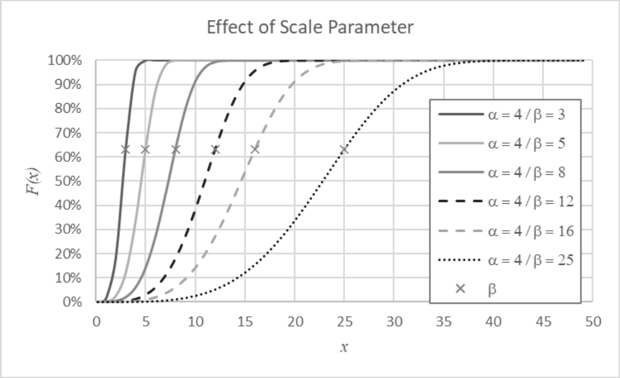
α = shape parameter





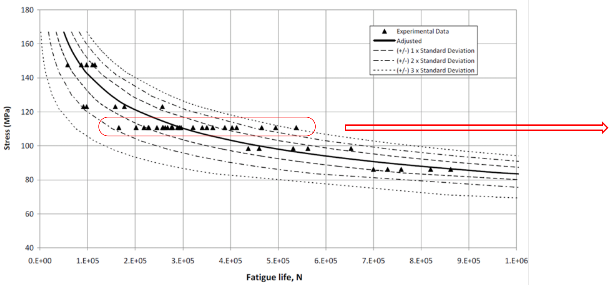
β = scale parameter

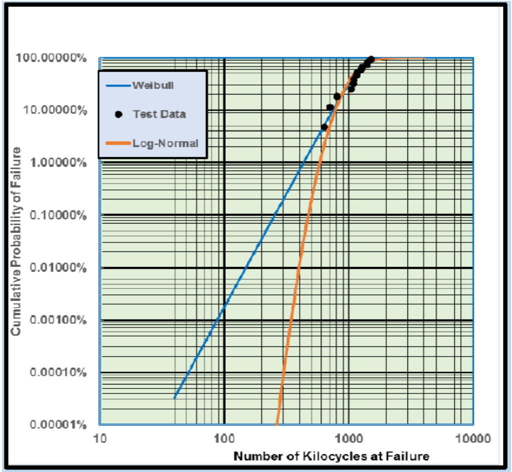




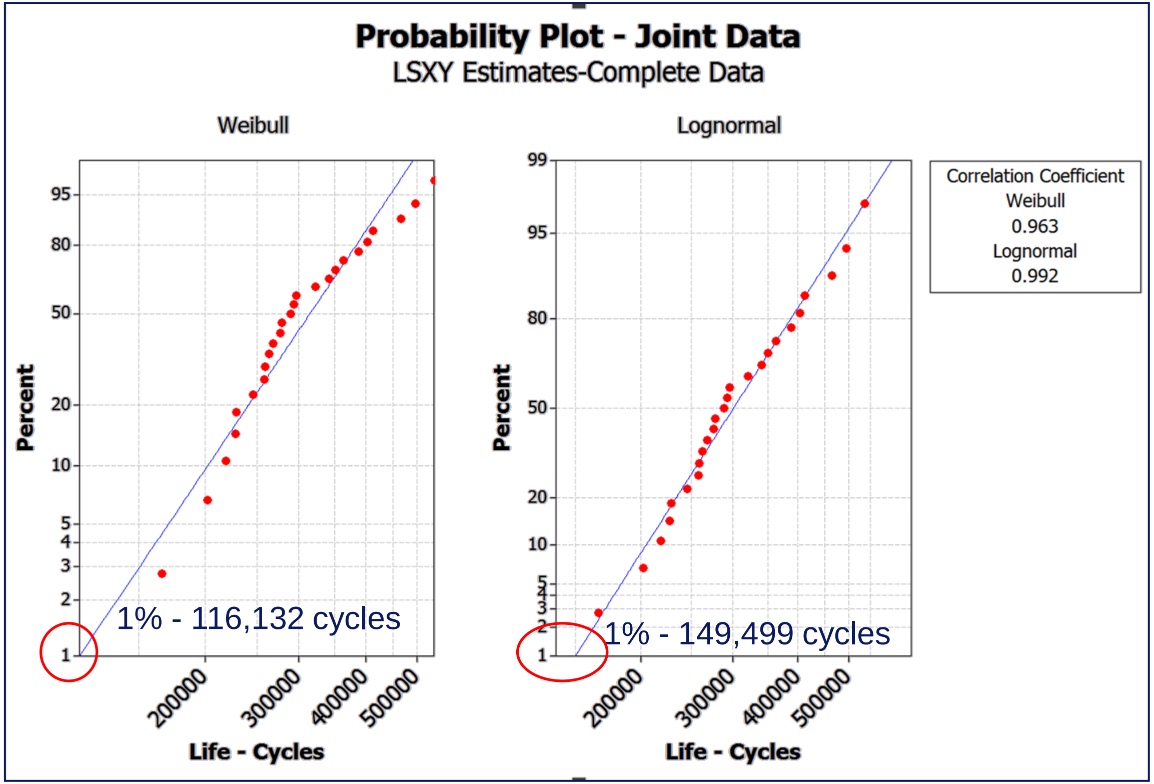
NOTE: Some Bibliography call the shape parameter β and the scale parameter η. Pay attention to avoid mistakes!

THE WEIBULL DISTRIBUTION





BROT, A., Three Faces of Aeronautical Fatigue – ICAF 2017



Weibull is always more conservative at low probabilities!

DEVELOPING THE KNOCK-DOWN FACTORS STATISTICALLY

THE RELIABILITY FACTOR

Is used to convert mean or **characteristic life** data into a **reliable life** value. Using a **Weibull** model and **common shape factors** the factors presented in the Table below can be applied.

Material	95% Reliability	99% Reliability
Aluminium	0.48	0.32
Titanium and Steel (<200 Ksi)	0.37	0.22
Steel (≥ 200 Ksi)	0.26	0.12

$$R(x) = e^{-\left(\frac{x}{\beta}\right)^{\alpha}}$$

$$ln[R(x)] = \left[-\left(\frac{x}{\beta}\right)^{\alpha}\right]$$

$$\left(\frac{x}{\beta}\right)^{\alpha} = -ln(R_x)$$

$$Reliability\ Factor = \frac{x}{\beta} = [-ln(R_x)]^{1/\alpha}$$

Material	α	*95% Reliability*	*99% Reliability*
Aluminium	4.0	0.476	0.317
Titanium and Steel (<200 ksi)	3.0	0.372	0.216
Steel (>200 ksi)	2.2	0.259	0.124

DEVELOPING THE KNOCK-DOWN FACTORS STATISTICALLY

THE CONFIDENCE FACTOR

Is a statistical-based value, which **addresses the uncertainty** associated with the final design value **caused by the limited test sample size**. For small specimens and common materials, a value of **0.7** for the standard **95% lower confidence bound** across the board shall be used. This value is slightly conservative for aluminum alloys and is a moderate fit for high-strength steels.

4. LOWER BOUND INTERVAL ESTIMATE OF FATIGUE-LIFE DISTRIBUTION

$$\breve{\beta}_\gamma = \hat{\beta}\left[\frac{1}{2n_f}\,\chi^2_\gamma\,(2n_f)\right]^{-1/\alpha}$$

Source: AFML-TR-69-65 / Whittaker and Besuner

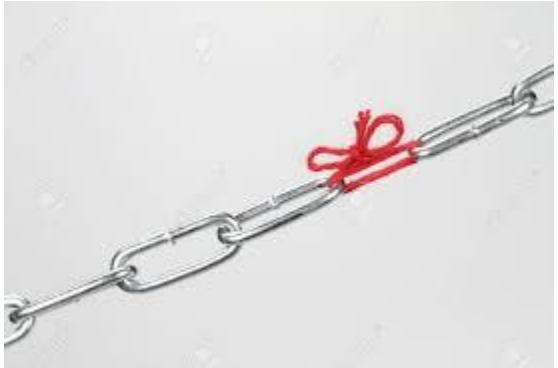
$$Confidence\ Factor = \frac{\breve{\beta}_\gamma}{\hat{\beta}} = \left[\frac{2n_f}{\chi^2_{\gamma(2n_f)}}\right]^{1/\alpha}$$

Material	α	*Confidence Factor*
Aluminium	4.0	0.76
Titanium and Steel (<200 ksi)	3.0	0.69
Steel (>200 ksi)	2.2	0.61

DEVELOPING THE KNOCK-DOWN FACTORS STATISTICALLY

THE SCALE FACTOR

Adjusts the design life based on the **ratio of details in the specimen to the number of details in the actual structure**. Using Weibull statistics, this factor is determined and the resulting values are tabulated.

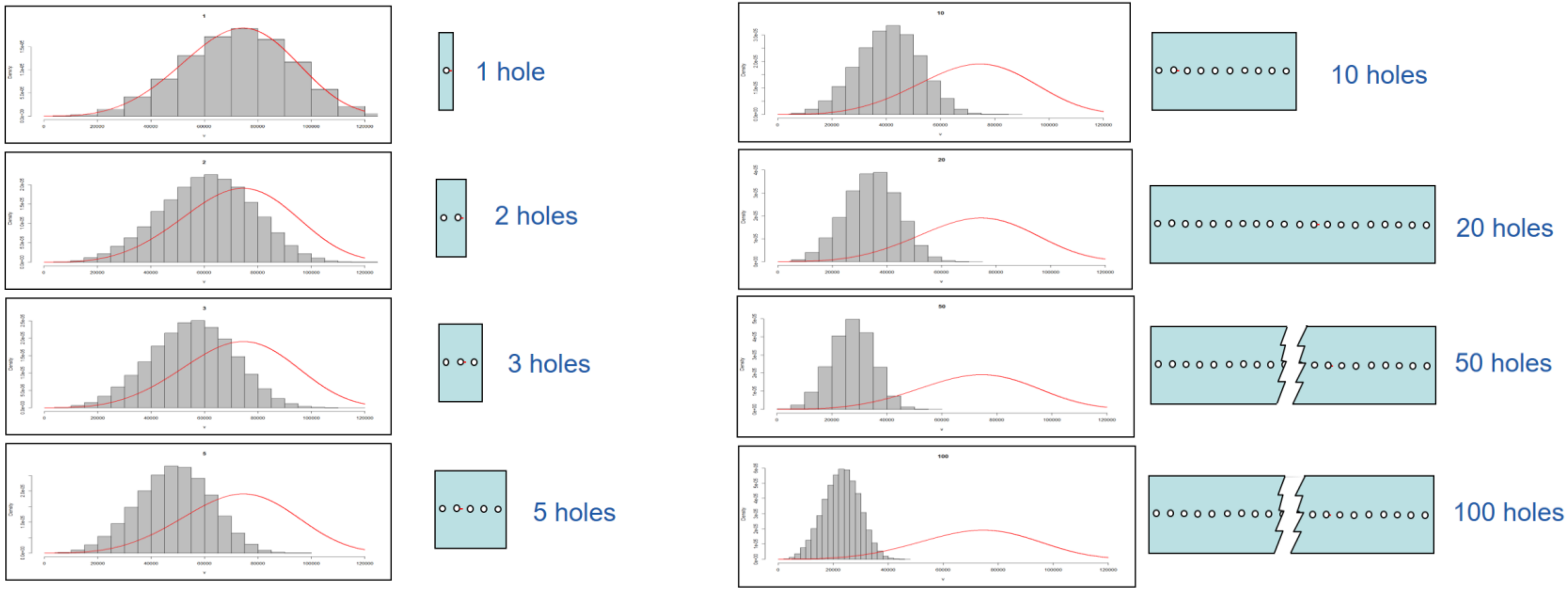


- "A chain is only as strong as its **weakest link**"
- What if there is no "weakest link", and **all links have the same probability** of failure?
- The Reliability of the "chain" for a given *x* is a product of the Reliability of their *n* links at the same *x*:

$$R(x)_{chain} = \prod_{link=1}^{n} R(x)_{links} = (R(x)_{links})^n$$

DEVELOPING THE KNOCK-DOWN FACTORS STATISTICALLY

THE SCALE FACTOR – NUMERICAL SIMULATIONS



DEVELOPING THE KNOCK-DOWN FACTORS STATISTICALLY

THE SCALE FACTOR

$$R(x)_{link} = e^{-(x/\beta)^{\alpha}}$$

$$R(x)_{chain} = \prod_{link=1}^{n} e^{-(x/\beta)^{\alpha}} = e^{-n.(x/\beta)^{\alpha}}$$

$$\ln[R(x)_{chain}] = \ln\left[e^{-n.(x/\beta_{link})^{\alpha}}\right]$$

$$-n.\left(\frac{x}{\beta_{link}}\right)^{\alpha} = \ln[R(x)_{chain}]$$

$$\ln\left(\frac{x}{\beta_{link}}\right)^{\alpha} = \ln\left[-\frac{\ln[R(x)_{chain}]}{n}\right]$$

$$\ln\left(\frac{x}{\beta_{link}}\right) = \frac{1}{\alpha}.\ln\left[-\frac{\ln[R(x)_{chain}]}{n}\right]$$

$$x_{chain} = \beta_{link}.e^{\left[\frac{1}{\alpha}\ln\left[-\frac{\ln(1-f(x))}{n}\right]\right]}$$

$$scale = \frac{x_{chain}}{x_{link}}$$

$$scale = \frac{\beta.e^{\left(\frac{1}{\alpha}\ln\left(-\ln\left(1-f(x)\right)\right)\right)}}{\beta.e^{\left(\frac{1}{\alpha}\ln\left(-\frac{\ln(1-f(x))}{n}\right)\right)}}$$

$$scale = e^{\left[\left\{\frac{1}{\alpha}\ln\left[-\ln\left(1-f(x)\right)\right]\right\}-\left\{\frac{1}{\alpha}\ln\left[-\frac{\ln(1-f(x))}{n}\right]\right\}\right]}$$

$$scale = e^{\left[\left\{\frac{1}{\alpha}\ln\left[-\ln\left(1-f(x)\right)\right]\right\}-\left\{\frac{1}{\alpha}\ln\left[-\ln\left(1-f(x)\right)\right]\right\}-\left\{\frac{1}{\alpha}\ln[n]\right\}\right]}$$

$$\boxed{scale = e^{\left(-\frac{1}{\alpha}\ln(n)\right)}}$$

- Not dependent on β
- Not dependent on f(x)
- **Depends on α and n**

DEVELOPING THE KNOCK-DOWN FACTORS STATISTICALLY

THE SCALE FACTOR

Percent of structure represented in test	Aluminium	Titanium & Steel < 200 Ksi	Steel ≥ 200 Ksi
≥ 100	1	1	1
50-100	0.84	0.79	0.73
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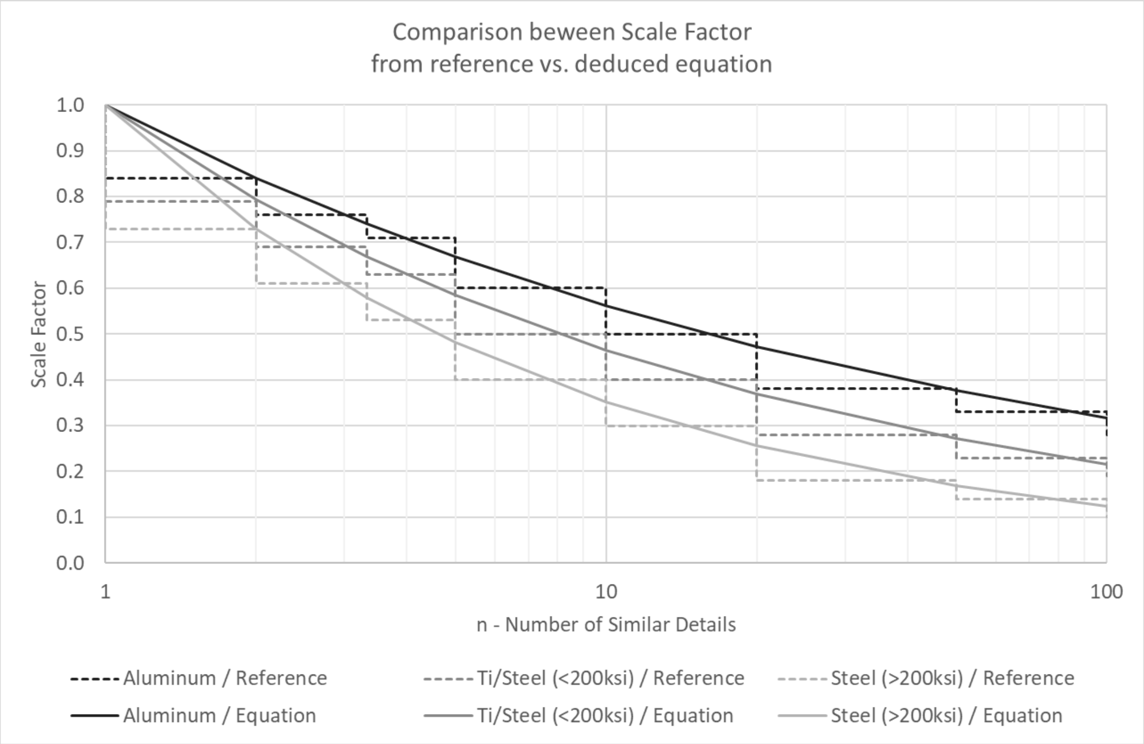
Less than 1%	0.28	0.19	0.1

$$scale = e^{\left(-\frac{1}{\alpha}\ln(n)\right)}$$

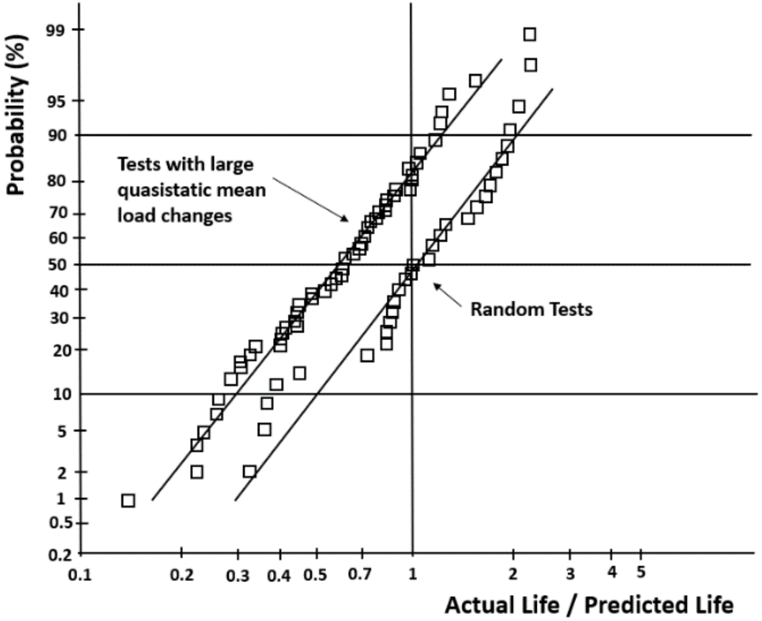


DEVELOPING THE KNOCK-DOWN FACTORS STATISTICALLY

THE TESTING FACTOR

Conservatively accounts for differences in the scale and fidelity of the test, including the extent to which the **loading of the test article replicates the actual structure**. When small coupons are used, a value of **0.7** shall be applied;

- The only factor that has **not a statistical origin**
- Variable amplitude loading / Pålmgren-Miner rule
- According to Schijve, certain shortcomings of the rule must be understood.
- The Relative Miner rule, $\sum n/N = q$, with $q < 1$ selected by experience.



Adapted from Schijve and Schutz

The authors of the present work believe that testing factor introduced by Safarian is a **correction to the Miner rule**, and may have the same meaning of the q proposed by Schijve at his Relative Miner Rule.

The present work does not intend to draw additional or final conclusions on this factor, but recommends that such shortcomings of the Miner rule have to be **further investigated and better understood**.

EXAMPLES AND DISCUSSION

THE TYPICAL SCATTER FACTOR FOR FATIGUE

For a single detail, made out of aluminum, with reliability/confidence equal to 95/95:

- Reliability Factor (95%): 0.48
- Confidence Factor (95%): 0.7
- Scale Factor (single detail): 1.0
- Testing Factor: 0.7

$$N_{95/95} = RF \cdot CF \cdot SF \cdot TF \cdot \beta = 0.48 \cdot 0.7 \cdot 1.0 \cdot 0.7 \cdot \beta = 0.24\beta$$

$$RF_{50} = N_{50}/\beta = [-ln(0.5)]^{1/4} = 0.912$$

$$N_{50} = 0.912\beta \rightarrow \beta = N_{50}/0.912$$

$$N_{95/95} = (0.24/0.912).N_{50} \approx N_{50}/4$$

EXAMPLES AND DISCUSSION

CALCULATING AN INTERMEDIATE SCALE FACTOR

The equation for the Scale Factor can replace the knock-down factors by intermediate values.

- Aluminum structure, 6 times more features than the arrangement tested.
- 1/6, or 16.7% of the structure represented on the test.
- From the tables, in the range of 10% to 20%, the scale factor equals to 0.6.
- $scale = e^{\left(-\frac{1}{\alpha}\ln(n)\right)} = e^{\left(-\frac{1}{4}\ln(6)\right)} = 0.64$
- A difference higher than 5% in comparison to the value from the Tables

This example leads to the conclusion that the **equation** for the Scale Factor can provide **more precise values** than the tabulated ones.

EXAMPLES AND DISCUSSION

CALCULATING THE KNOCK DOWN FACTORS FOR A DIFFERENT ALFA

Item 417 from Whittaker:

- 31 test articles
- Aluminum 7075-T6, notched
- Spectrum loaded
- Weibull shape parameter: $\alpha = 7.4119$

Supposing 20 details, or 5% represented in the test

	Tables	Calculated
Reliability Factor (99%)	0.32	0.54
Confidence Factor (γ = 95%, n = 31)	0.70	0.96
Scale Factor (n = 20)	0.50	0.67
Testing Factor	0.70	1.00
Knock down	**0.078**	**0.35**

approx 5x

EXAMPLES AND DISCUSSION

OBTAINING THE KNOCK-DOWN FACTORS FOR DIFFERENT DISTRIBUTIONS

Often the design focus on the reliability of a detail given the target reliability for the assembly.

$$R(x)_{assembly} = (R(x)_{detail})^n \qquad R(x)_{detail} = \left(R(x)_{assembly}\right)^{\frac{1}{n}}$$

While for Weibull we can solve analytically, for some distributions this can be made only **numerically**, but we can use a **similar rationale**.

Weibull

$$Combined\ Factor|_{RF+SF} = \frac{x}{\beta} = \left[-ln\left(\left(R(x)_{assembly}\right)^{\frac{1}{n}}\right)\right]^{1/\alpha}$$

$$Combined\ Factor|_{RF+SF} = \frac{x}{\beta} = \left[-ln\left((0.99)^{\frac{1}{100}}\right)\right]^{1/4} = 0.10$$

log-Normal

$$\log x|_{assembly,R99\%} = NORM.INV[(1-0.9999); \log 10^6; 0.14] = 5.4795$$

$$Combined\ Factor|_{RF+SF} = \frac{x_{assembly,99\%}}{\mu} = \frac{10^{5.4795}}{10^6} = \frac{301{,}657}{1{,}000{,}000} = 0.3$$

CONCLUSIONS AND COMMENTS

This work brought an **in-depth investigation** on the knock-down factors proposed by Safarian in Jerusalem, 2013, presenting a **background** and **deduction** or an **explanation** for them based on **statistical approaches**.

- The Reliability and the Confidence Factors convert the mean or average values into values that meet the higher reliability targets, and account for the uncertainties due to the reduced test samples.
- The Testing Factor has no statistical origin, being more related to the shortcomings of the Pålmgren-Miner rule, and has to be further investigated and understood.
- The Scale Factor is similar to require higher reliability levels, and may be overconservative based on the statistical distribution chosen.

CONCLUSIONS AND COMMENTS

The Scale Factor is derived from the assumption that the **failure of an entire assembly** occurs when its **first individual element presents a crack** nucleation.

This can be **overconservative** because it does not account for

- crack growth;
- crack interactions;
- load redistribution;
- other effects on the actual structure.

Such conservativeness can be further removed by **testing structural arrangements as close as possible to the actual structure**.

CONCLUSIONS AND COMMENTS

By **understanding each and every factor**'s origin and built-in conservativeness, the engineer in charge can propose **alternative approaches** and come out with more **realistic results**, eventually saving weight and resources, and developing optimized structures **without compromising safety**.

QUESTIONS?

THANK YOU

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