Past, Present, and Future Stress Intensity Factor Solutions for Cracks at Holes

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Southwest Research Institute®

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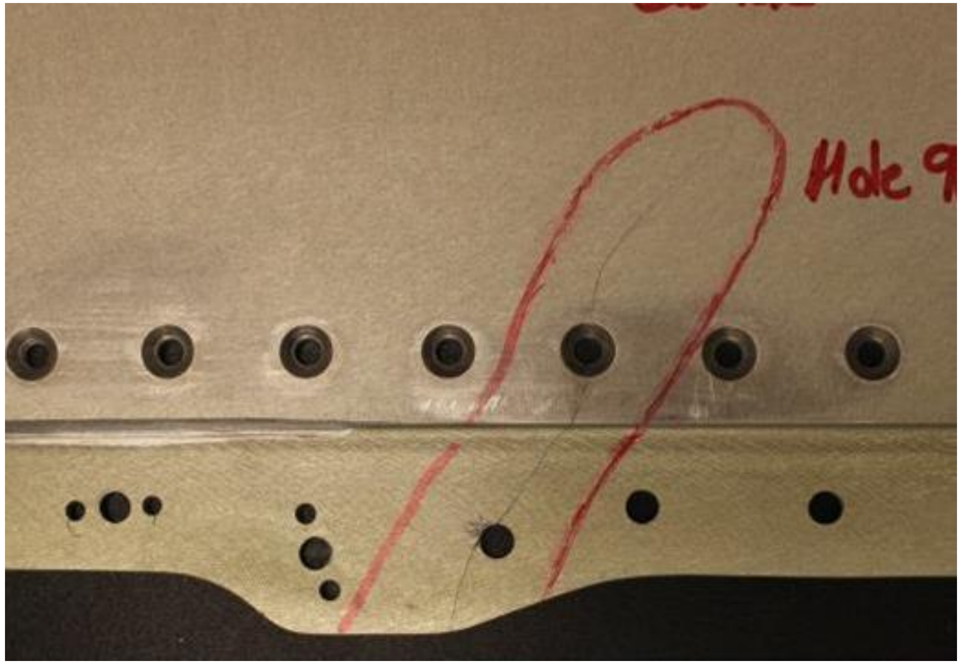
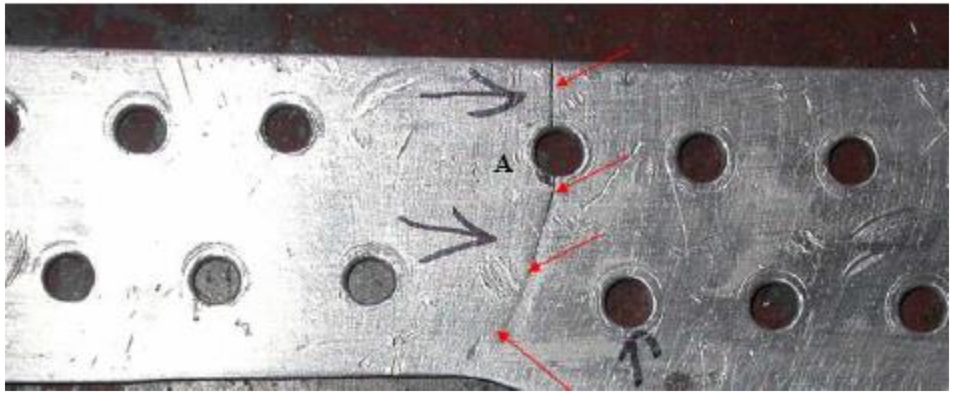
Holes are Common Sites for Fatigue Cracks

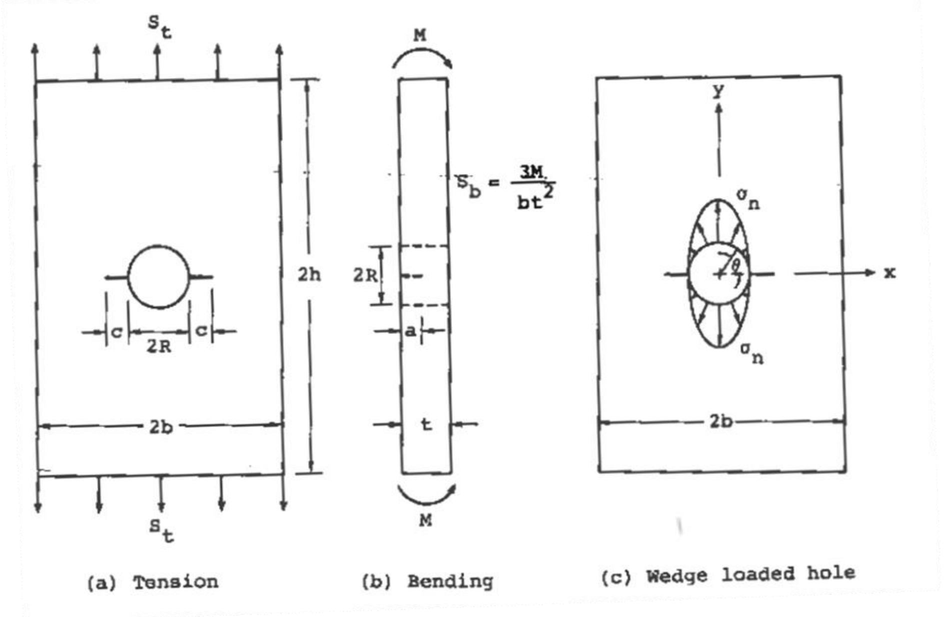


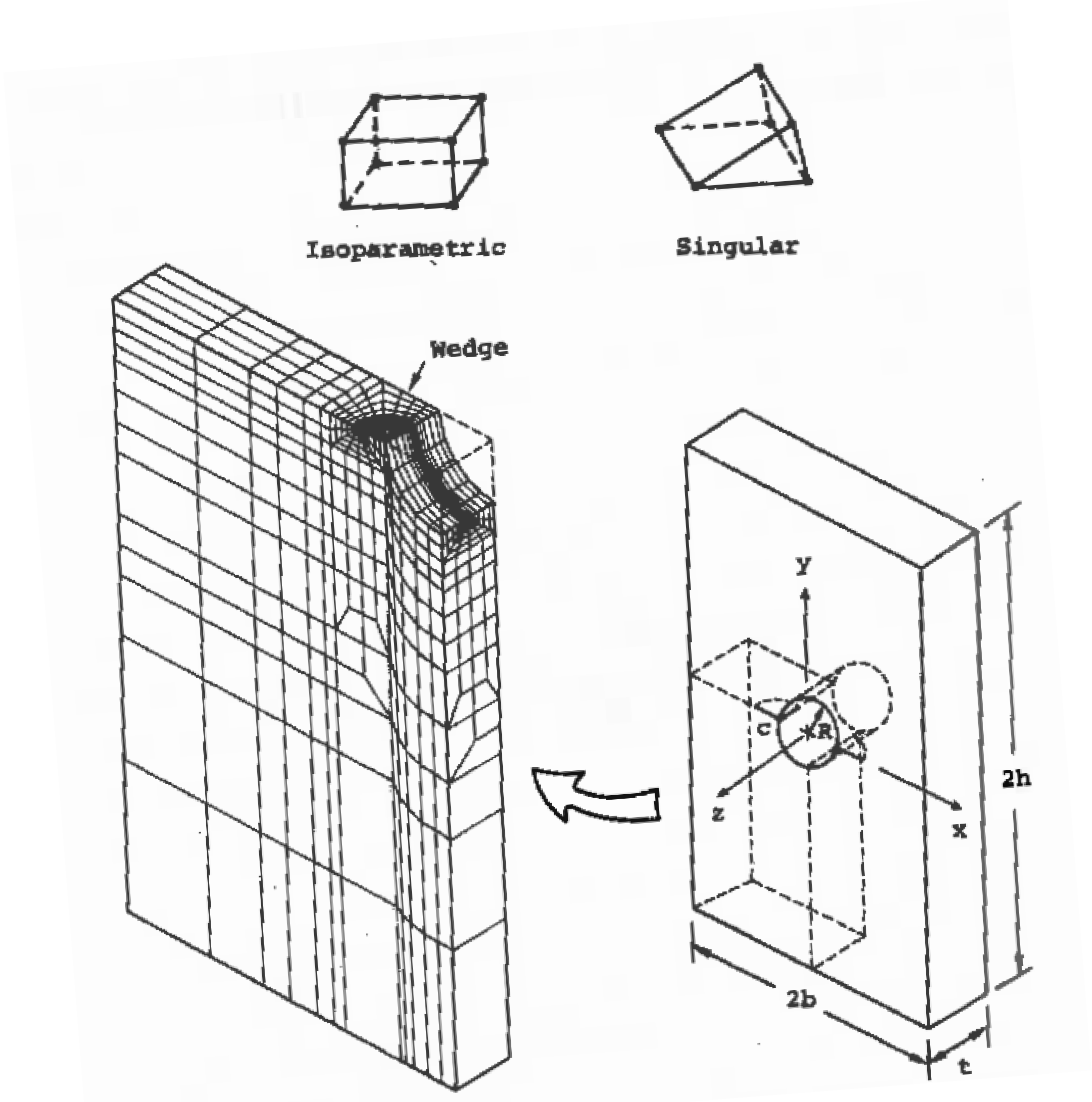


- Hole stress concentration increases the chance of fatigue cracks forming and growing there
- An airframe will have many holes and therefore many potential fatigue-critical locations
- Airframe certification may require damage tolerance analyses at large number of locations
- Practical stress intensity factor solutions needed that are accurate, fast, and robust
- Corner cracks especially important because they are usually initial crack sites and often dominate life

Newman & Raju Developed Landmark Solutions

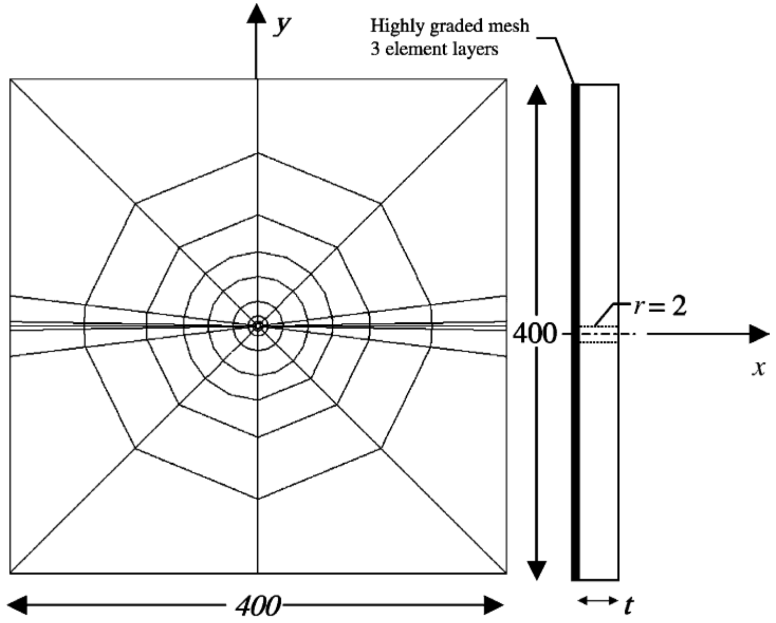
- Raju & Newman (1979) finite element solutions
  - Symmetric corner cracks at a hole
  - Wide plates with remote loading
  - 18 FE geometry models with 9300 DOF
- Newman & Raju (1983, 1986) closed-form equations
  - Include approximate finite-width corrections
  - Widely used for 35 years

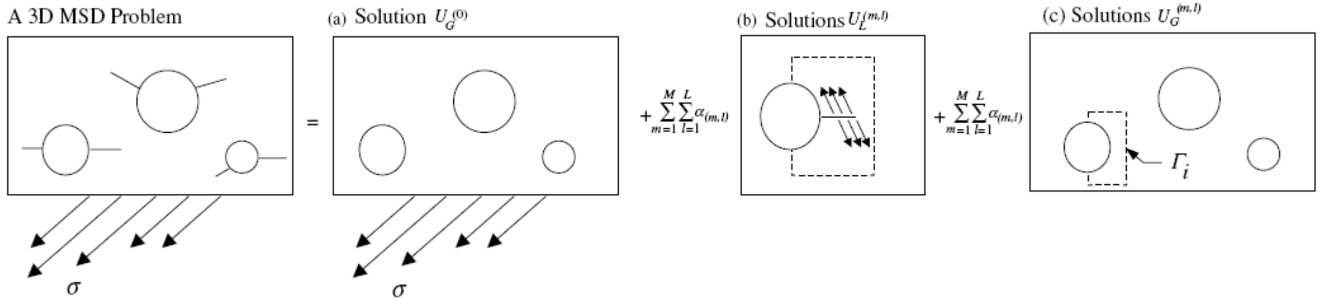




Fawaz & Andersson (2004) Generated the Next Landmark

- *p*-version FE method
- Single or double (dissimilar) corner cracks in wide plates with remote loading
- Large data tables
  - 7150 combinations of *R*/*t*, *a*/*t*, *a*/*c*
  - Millions of solutions built using a splitting scheme for non-symmetric corner cracks
- Generation of new data tables (corrections as well as different configurations) is continuing



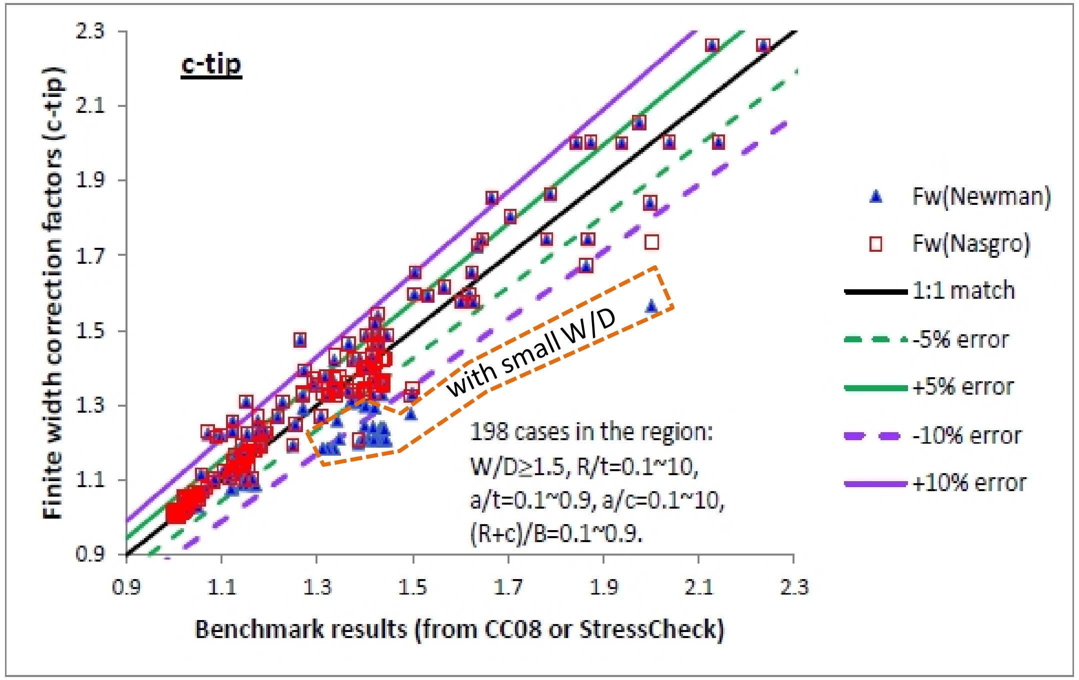


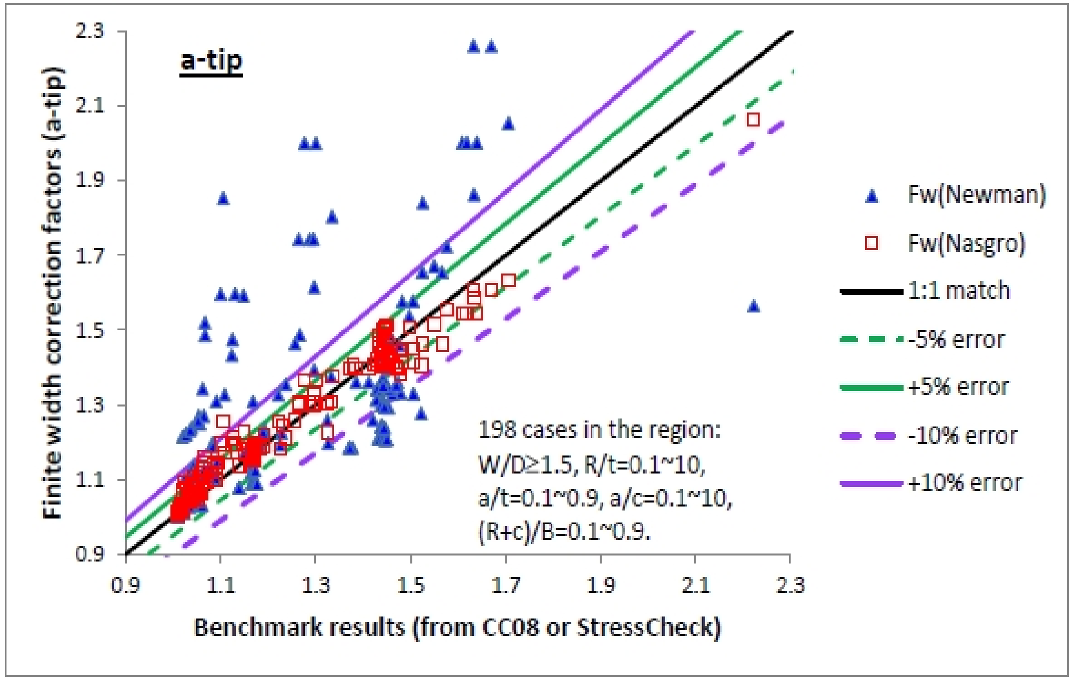
What are the Limitations of N-R and F-A?

- Practical realities of computing a solution
  - Idealized geometries with limited complexity
  - Full-factorial expansion of all degrees of freedom in the solution
  - Significant effort to build, process, and verify the solutions
  - Large computational costs associated with running the models (even today…)
- As a result, both solution sets share similar characteristics
  - Generated for infinite width plates (*i.e.*, tiny holes) and often employ multiple symmetries
  - Support a subset of uniform remote loadings: tension, bending, and pin loading
  - Linear-elastic material response
  - Assume an initial stress-free geometry
- Real holes in structures that need practical structural analysis…
  - Located in finite width plates and are offset from the centerline
  - Have stresses that deviate from the idealized remote loadings of tension, bending, and pin loading
  - May have localized plasticity near the edge of the hole
  - May have engineered residual stresses (*e.g.*, cold-expanded holes)

Finite-Width Corrections, Old and New

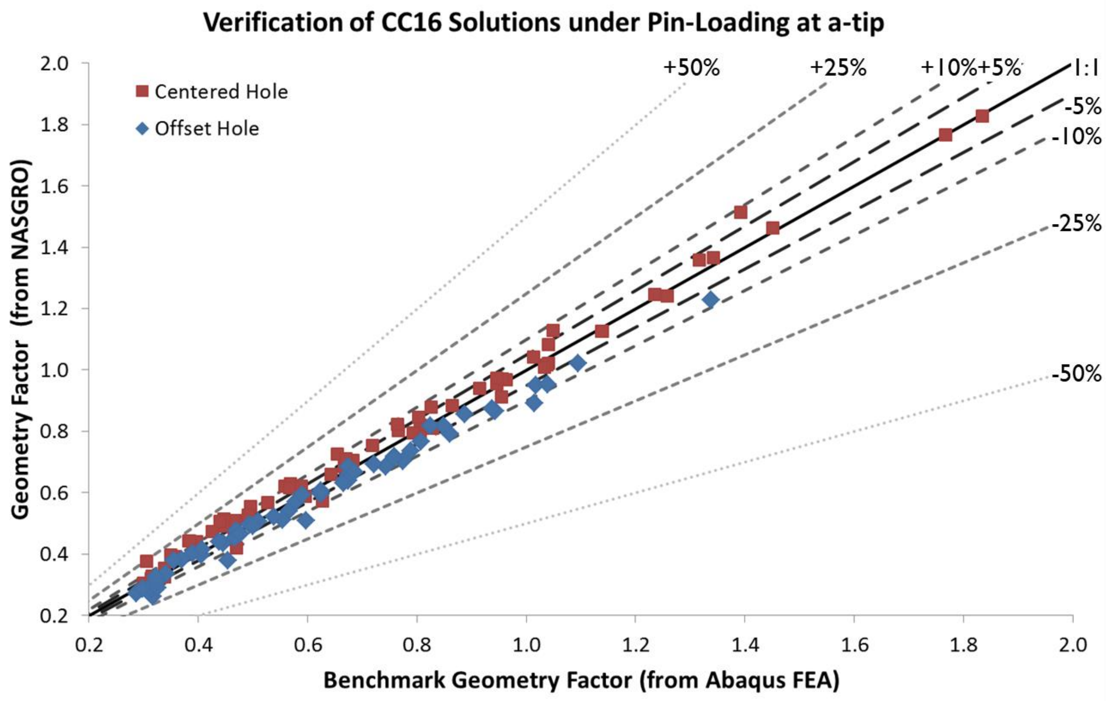
- Newman-Raju (1983, 1986) developed simple equations for finite-width corrections
- Guo developed improved equations (c2013)
- Models compared with benchmark solutions for 198 crack geometries (tension)

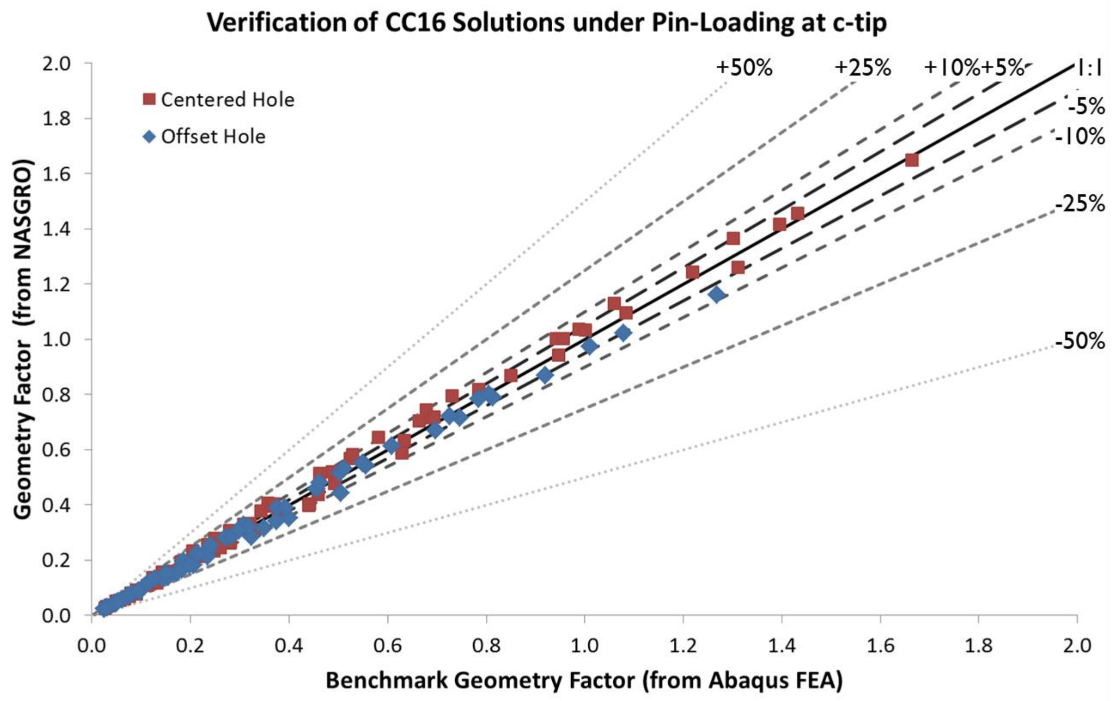




Finite-Width Corrections for Pin Loading

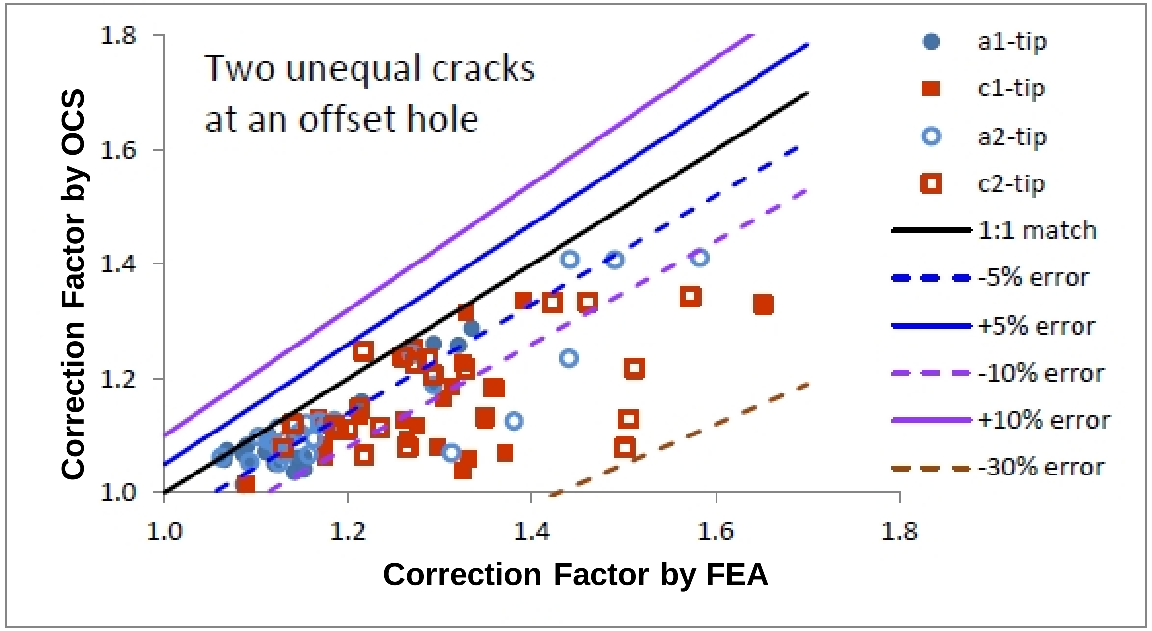
- Finite-width corrections for remote tension or bend are not accurate for pin loading
- New pin-loading correction factors (including ***both*** finite width ***and*** hole offset) developed from weight function solutions (Sobotka, c2017)

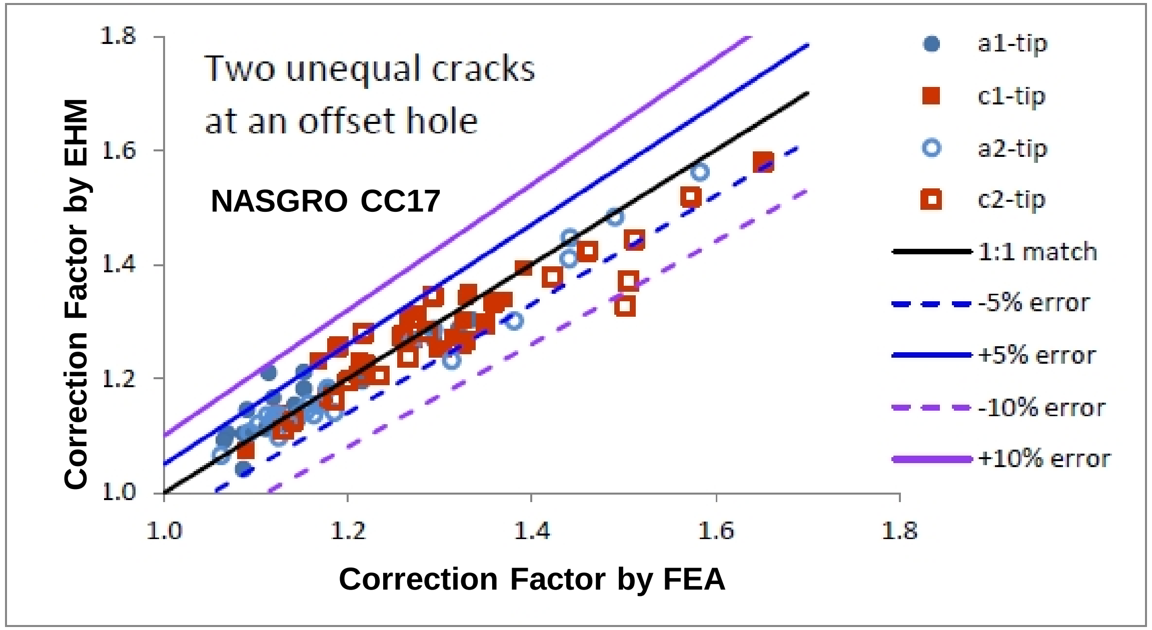




Finite-Width Corrections for Two Unequal Cracks

- "Equivalent Hole Method" (EHM) developed by Guo (c2013) for NASGRO CC17
- Compared here with "One Crack Solution" (OCS) ignoring the opposite crack
- Benchmarked against StressCheck solutions

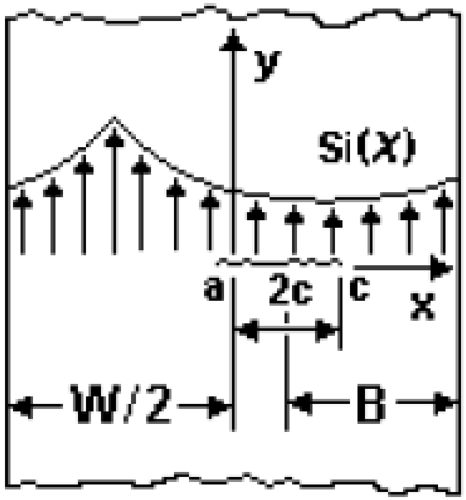




Note that correction factors inevitably contribute some error to the total solution

Weight Functions Provide a Different Approach

$$K_I = \int_{-c}^{+c} S(x) \times \boldsymbol{w}(\boldsymbol{x}) dx$$



- Integrate WF with stress distribution acting normal to crack plane in corresponding uncracked body
- WFs are based on reference solutions (high quality numerical SIF values for simple stress profiles and specific dimensions)
- Finite-width and hole-offset effects included in reference solutions
- Crack-plane stress distribution can be arbitrary (for example, include stress concentration effects as well as local residual stresses)
- WF approach is well-established and widely-used

Univariant WF Solution for Corner Crack at Hole

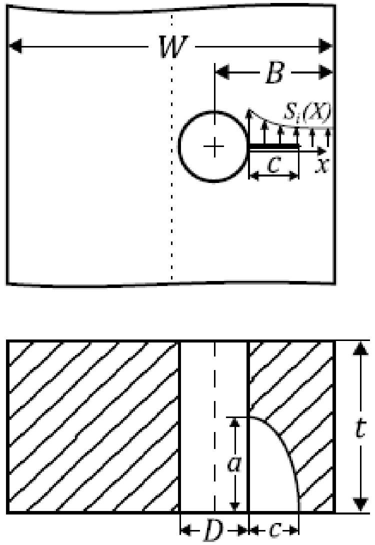
- Formulation by Lee (2003) based on Glinka approach (1991, 1998)

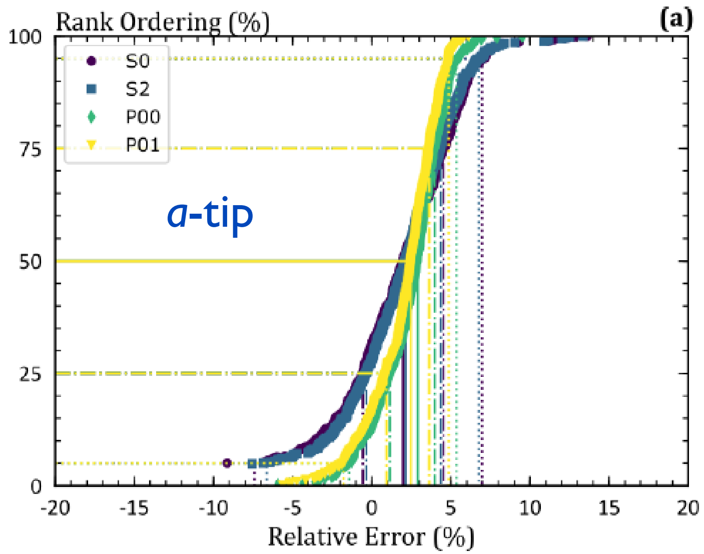
$$K_{a,c} = \int_0^c W_{a,c}\sigma(x)dx$$

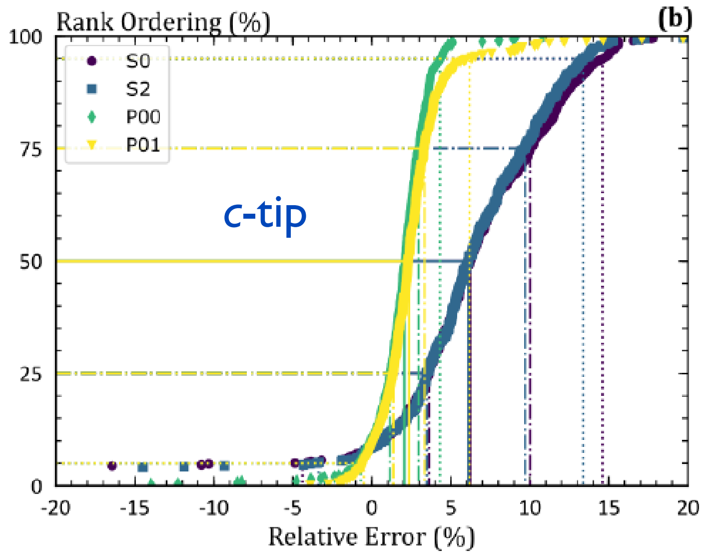
$$W_a = \frac{2}{\sqrt{\pi x}}\left[1 + M_{1a}\sqrt{\frac{x}{c}} + M_{2a}\frac{x}{c} + M_{3a}\left(\frac{x}{c}\right)^{3/2}\right] \qquad W_c = \frac{2}{\sqrt{2\pi(c-x)}}\left[1 + M_{1c}\sqrt{\frac{c-x}{c}} + M_{2c}\frac{c-x}{c} + M_{3c}\left(\frac{c-x}{c}\right)^{3/2}\right]$$

- Some solutions tend conservative because bivariant stresses are neglected

CC08





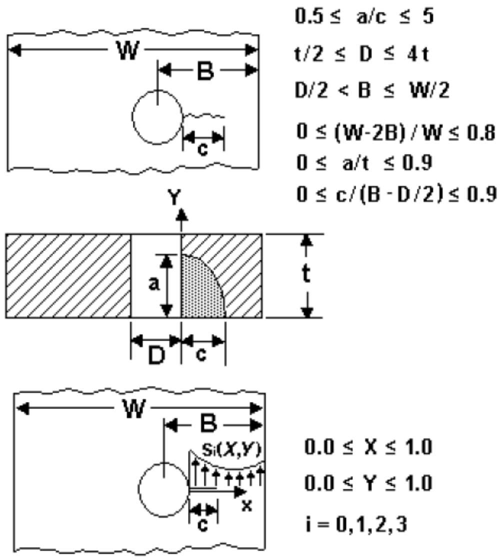


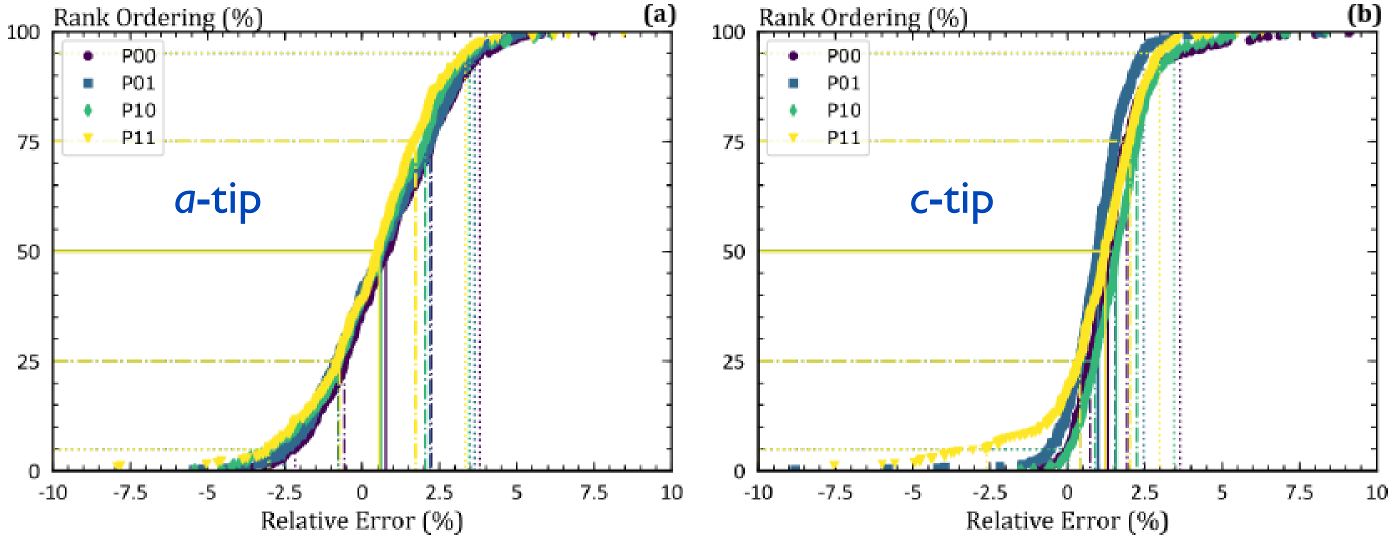
Bivariant WF Solution for Corner Crack at Hole

- Bivariant WF (CC10) developed by Lee (2004, 2008) based on Orynyak methods (1994, 1995)

$$K^{a,c} = \int_0^a \int_0^{c\sqrt{1-y^2/a^2}} \sigma(x,y) \frac{\sqrt{R^2 - r^2}}{\pi l^2_{QQ^{a,c}} \sqrt{\pi R}} \left(1 + \frac{l^2_{QQ^{a,c}}}{l^2_{\bar{Q}_x Q^{a,c}}} + \frac{l^2_{QQ^{a,c}}}{l^2_{\bar{Q}_y Q^{a,c}}}\right) \times$$
$$\times \left[1 + \Pi_1^{a,c} \sqrt{1 - \frac{r}{R}} + \Pi_2^{a,c} \left(1 - \frac{y}{y'}\right) + \Pi_3^{a,c} \left(1 - \frac{x}{x'}\right)\right] dx dy$$

- About 600 geometries in the revised CC26 reference solution matrix (Sobotka, c2022)

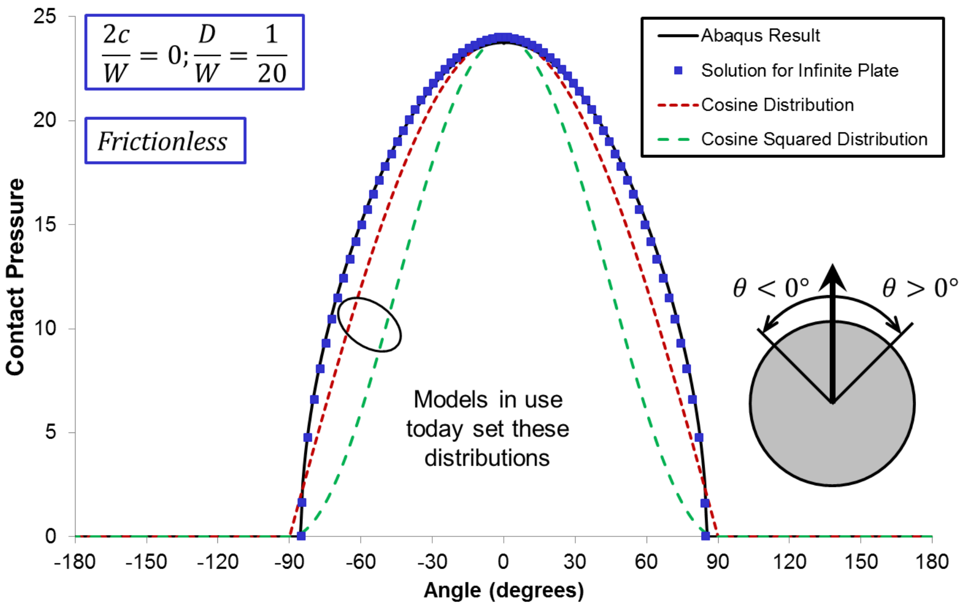




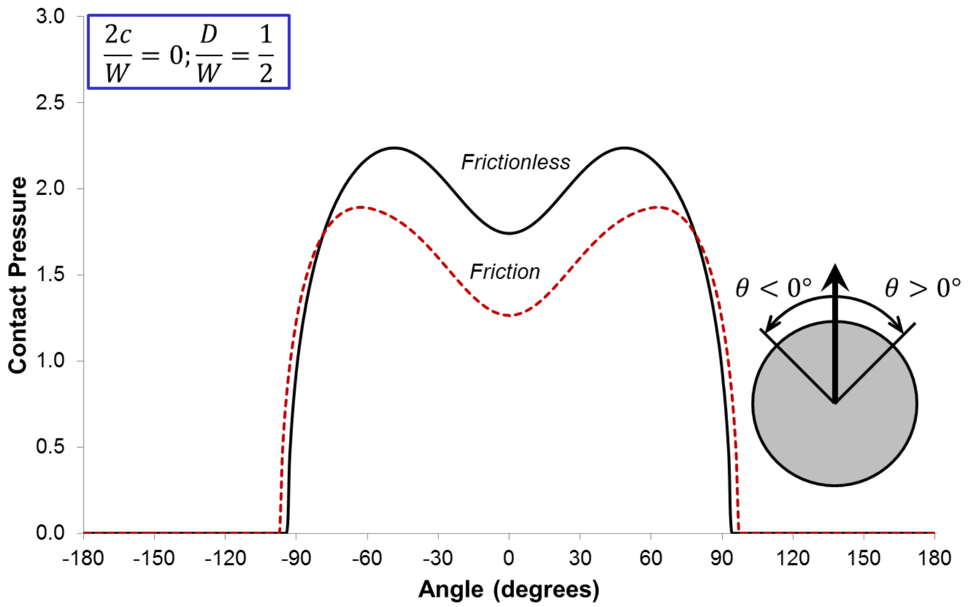
What about Pin Loading?

- FE analyses with contact and friction can be used to determine crack-plane stresses for a pin-loaded hole (Sobotka, c2014, 2020)
- These crack plane stresses can then be combined with weight functions to get SIF
- Contact pressures for finite plates not the same as idealized assumptions used in some classical solutions

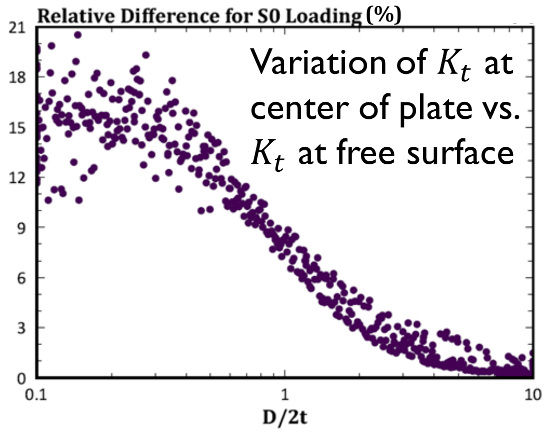
Wide Plate

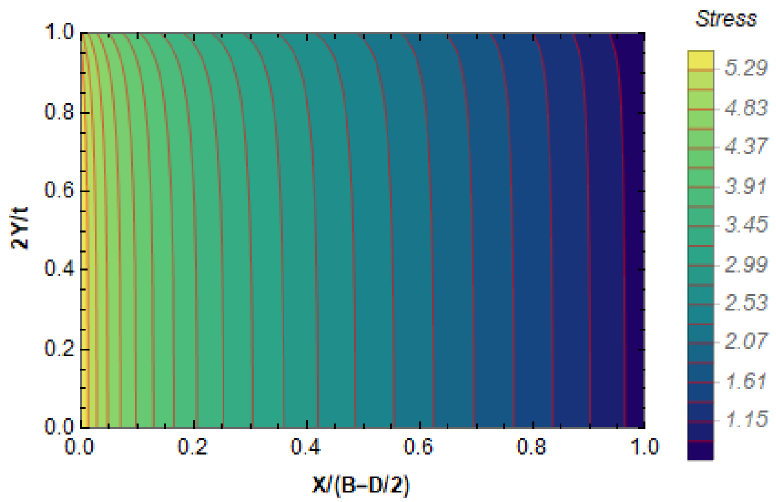


Narrow Plate



Can We Use WF Solutions with Remote Stresses?



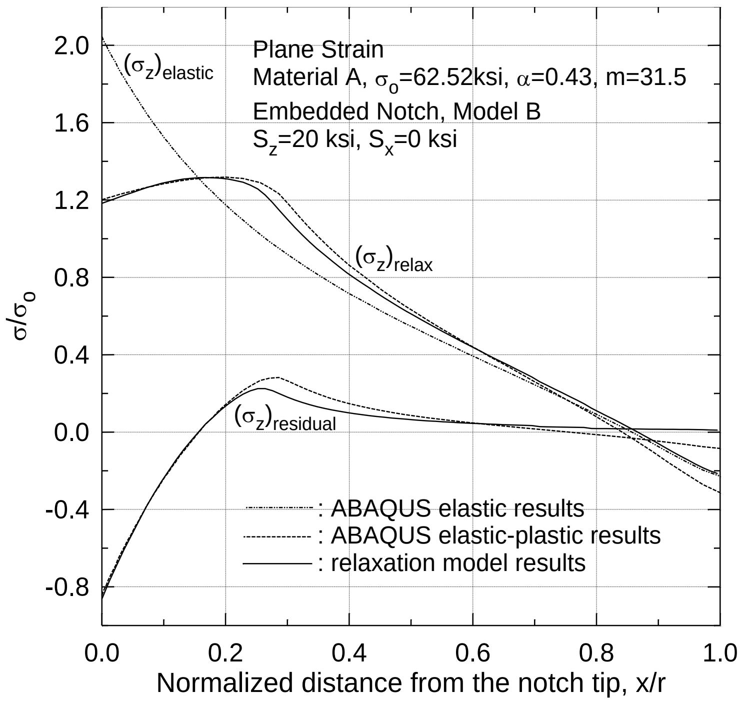


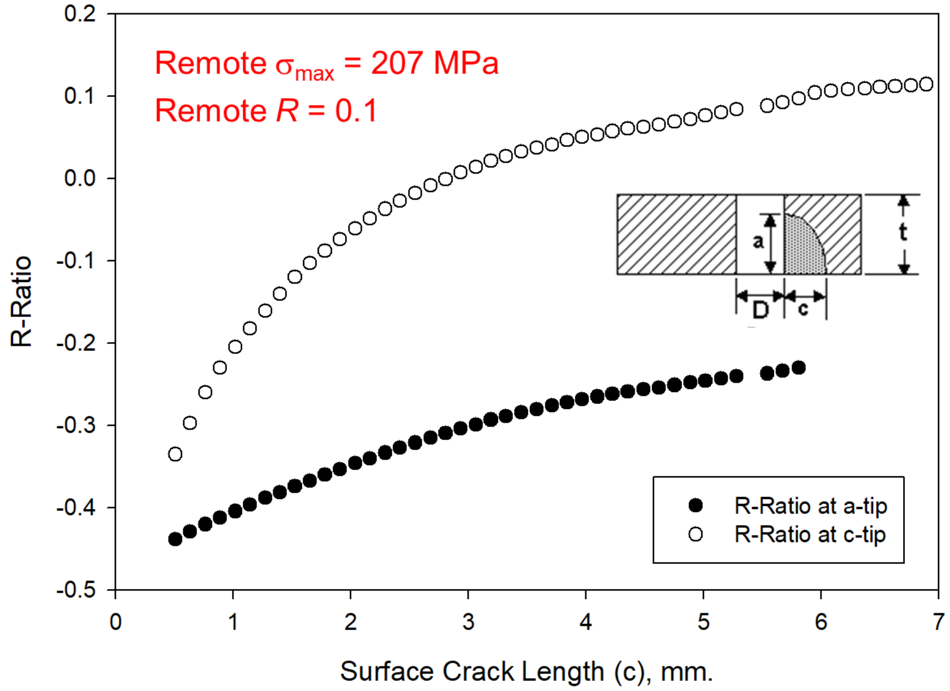
$$\boldsymbol{\sigma}(d_1, \ldots, d_g) \approx \boldsymbol{\mu} + \sum_{i=1}^{\bar{m}} c_i(d_1, \ldots, d_n) \times \boldsymbol{\phi}_i$$

- Yes! If we can accurately estimate the crack-plane stresses from the remote stresses
- Univariant stress gradient for hole in plate has been readily available (2D problem) for some time, but…
- Actual stress gradient on the crack plane is bivariant! (Up to a 20% difference in surface/interior stress concentration)
- New model for full bivariant (uncracked) stress gradient on the crack plane as a function of plate & hole geometry (DeCarlo, Sobotka, and Haikal, c2022)
  - Principal Component Analysis (PCA) to express stresses using a reduced set of 2D mode shapes
  - Modified Latin Hypercube Sampling (LHS) to determine FE solutions needed to calibrate the response surface model
  - Fit Gaussian Process (GP) regression models to predict Principal Component scores, used to reconstruct stress field
  - Verify with independent FE solutions

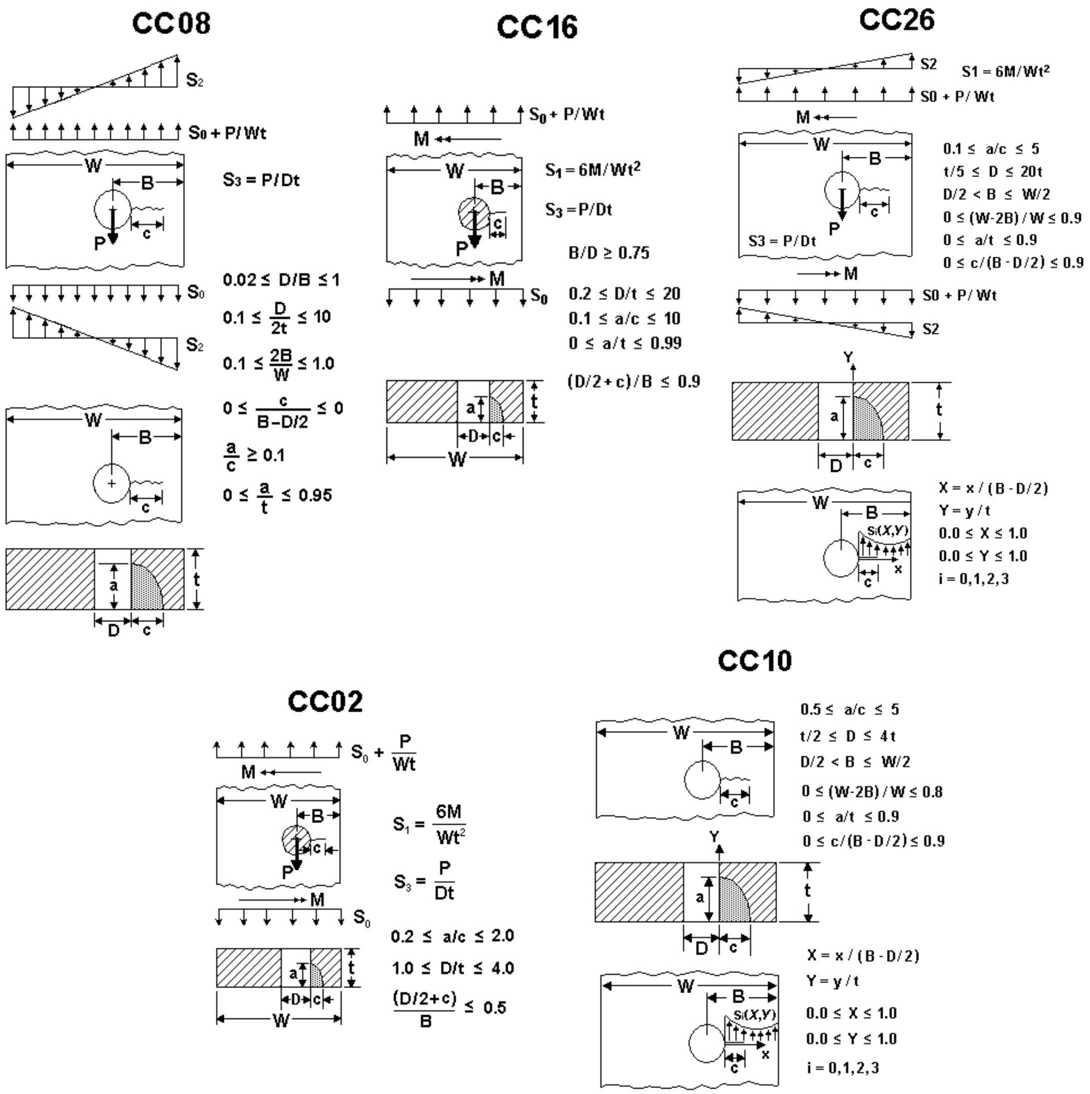
Local Plasticity (Shakedown) and Other Residual Stresses

- Yielding at hole edge will cause stress redistribution and relaxation
- Shakedown methods can be used to estimate local residual stresses
- Univariant and bivariant WF methods can include these and other RS (e.g., CX)
- Simple SIF superposition methods are usually adequate to characterize *da/dN* effects



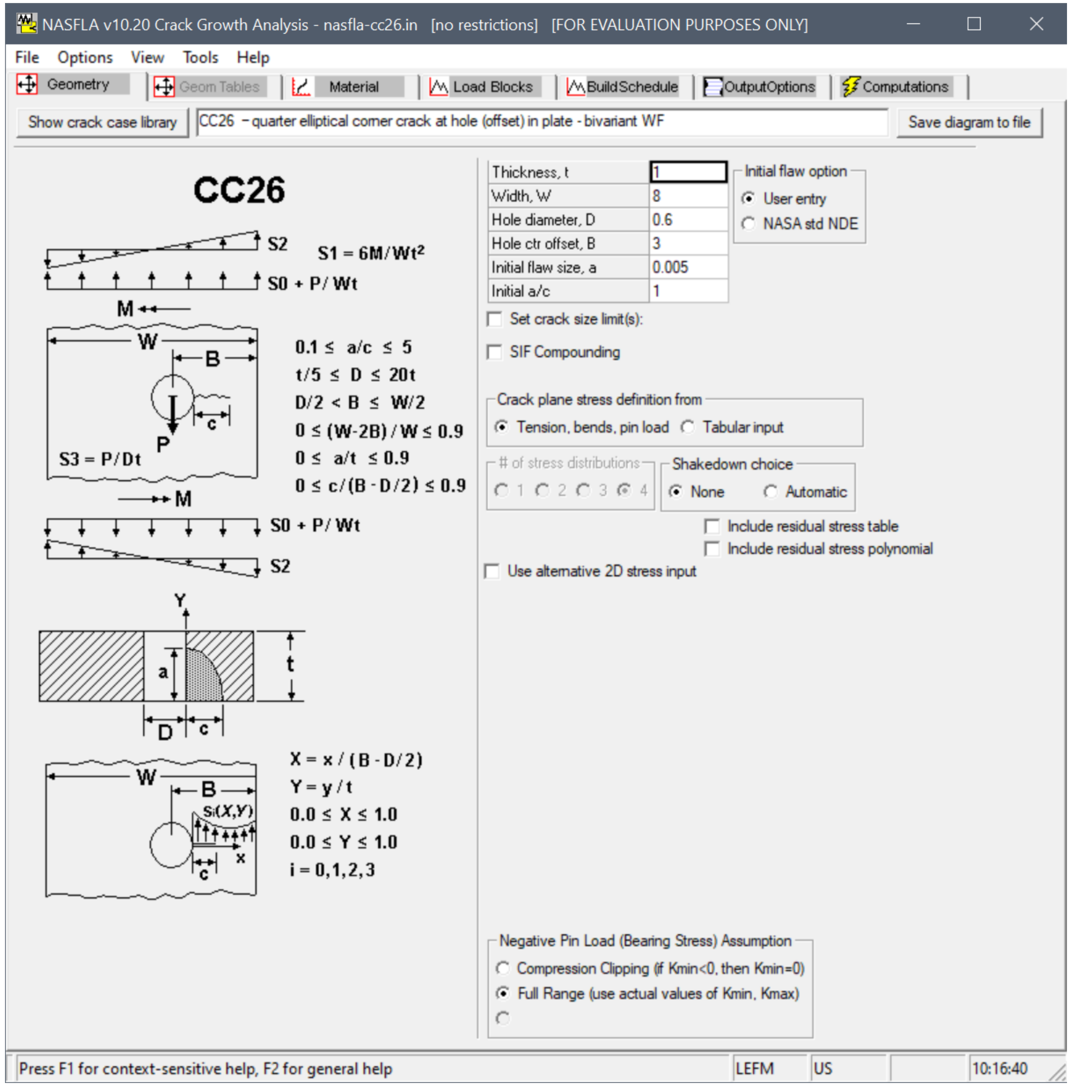


How do these Solutions Compare with Each Other?



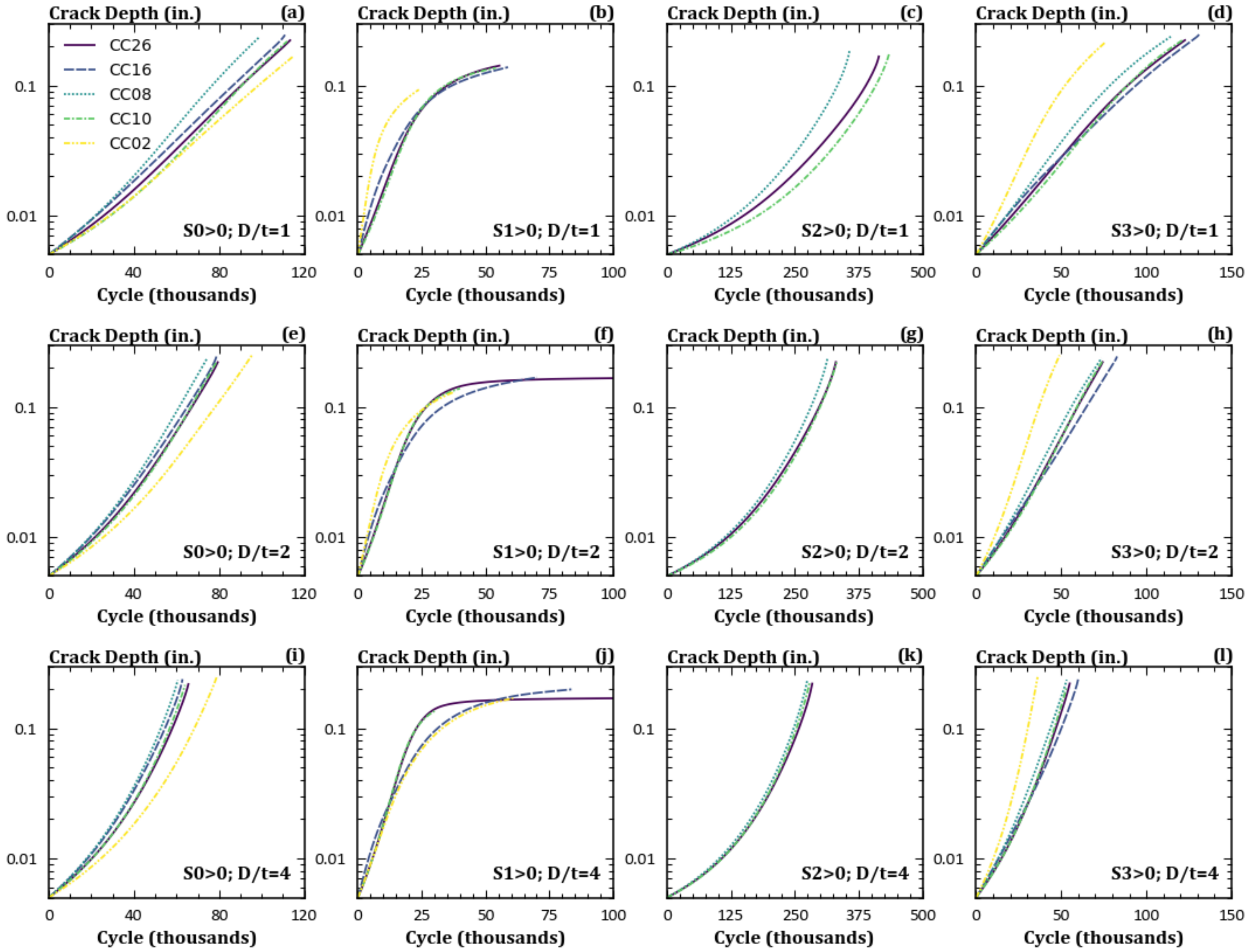
- NASGRO v10.2 has three solutions with wide geometric limits and that support remote loading
  - CC08 – Univariant weight function
  - CC16 – Fawaz-Andersson solutions
  - CC26 – Bivariant weight function
- Also considered here…
  - CC02 – Newman-Raju solutions now placed in the superseded category
  - CC10 – Previous generation of bivariant weight function solutions

Problem Statement



- Plate geometry case
  - Fixed: $D/2B = 0.25, 2B/W = 1$
  - Variable: $D/t = 1, 2, 4$
  - Thickness: $t = 0.25$“
- Initial crack: $a = 0.005$“ and $a = c$
- Material: 7075-T6 Plate (M7HA12AB1)
- Loading:
  - One remote loading is active at a time
  - Loading magnitude is fixed for geometry case and set to produce an “interesting life”
  - Loading spectrum is fixed at R=0.1 for all cases
- Results plot the corner crack lives and ignore contributions post-transition

Crack Depth vs. Cycles

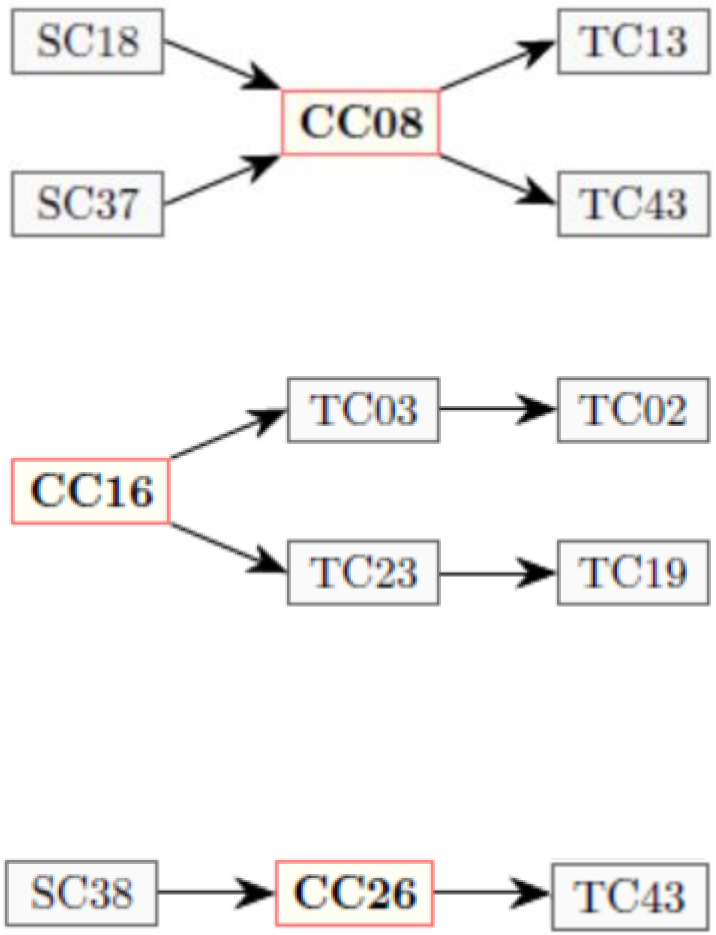


- Most solutions have lives within 2X of the mean predicted life
- CC02 (Newman-Raju) often is an outlier with lives that are too low or too high
- Predicted lives increasingly align as D/t increases, except for $S_1 > 0$ loadings
- CC26 agrees with consensus lives except for $S_1 > 0$ loading

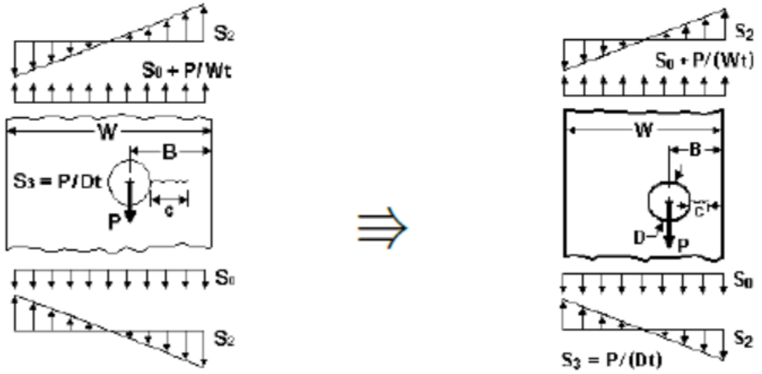
Additional Considerations of the DT Assessment

- Solving the initial corner crack at a hole problem in the idealized geometry is likely not the end of the life assessment
- There are more considerations in the DT assessment
  - What happens after the crack transitions from the original geometry?
  - What happens if the hole is in a row of holes?
  - What happens if there's out-of-plane bending post-transition?
  - What about countersunk holes?
  - What happens if the crack is at a hole in a lug?
  - What about interference fit?
  - How accurate are these solutions?
  - …

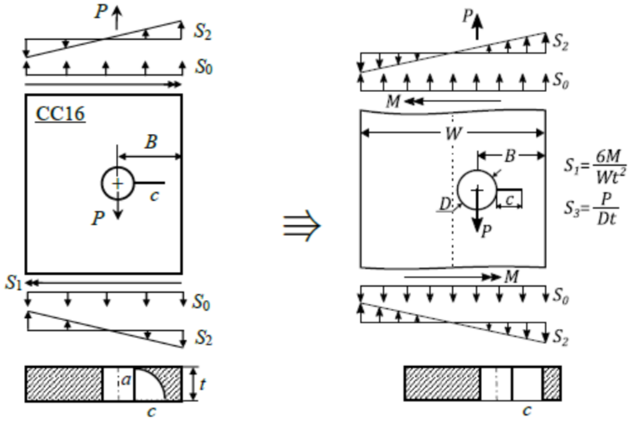
What Happens After *a* Corner Crack Transitions?



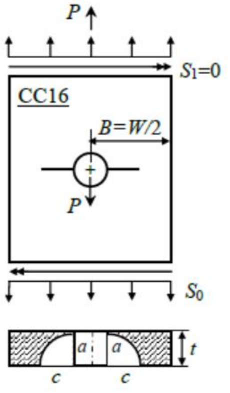
Post-transition routines map the corner crack into a straight through crack.
Ensure that weight functions map to weight functions, and so on…
Some remote loading terms may be lost.



(a) CC08 (b) TC43



(a) CC16 (b) TC03

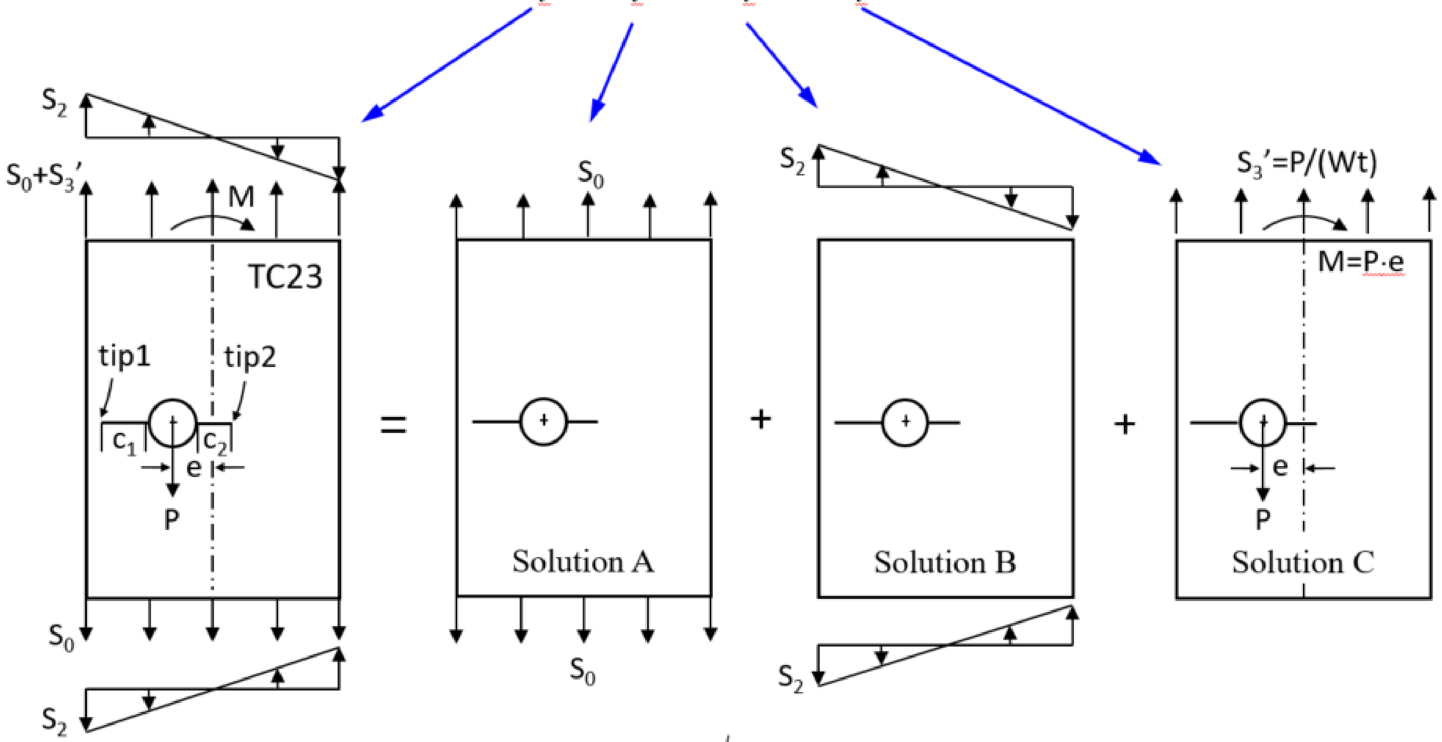


(a) CC16 (b) TC23

What Happens After Corner Cracks Transition?

- Two dissimilar corner cracks may eventually transition to dissimilar through cracks
- Compounding solution developed by Bombardier and Liao, NRC-Canada (2009, 2010) for tension, bend, and pin loading
- Improvements by Guo (c2019-20)

$$K_i = K_i^A + K_i^B + K_i^C \qquad i = 1, 2 \text{ (crack tip index)}$$



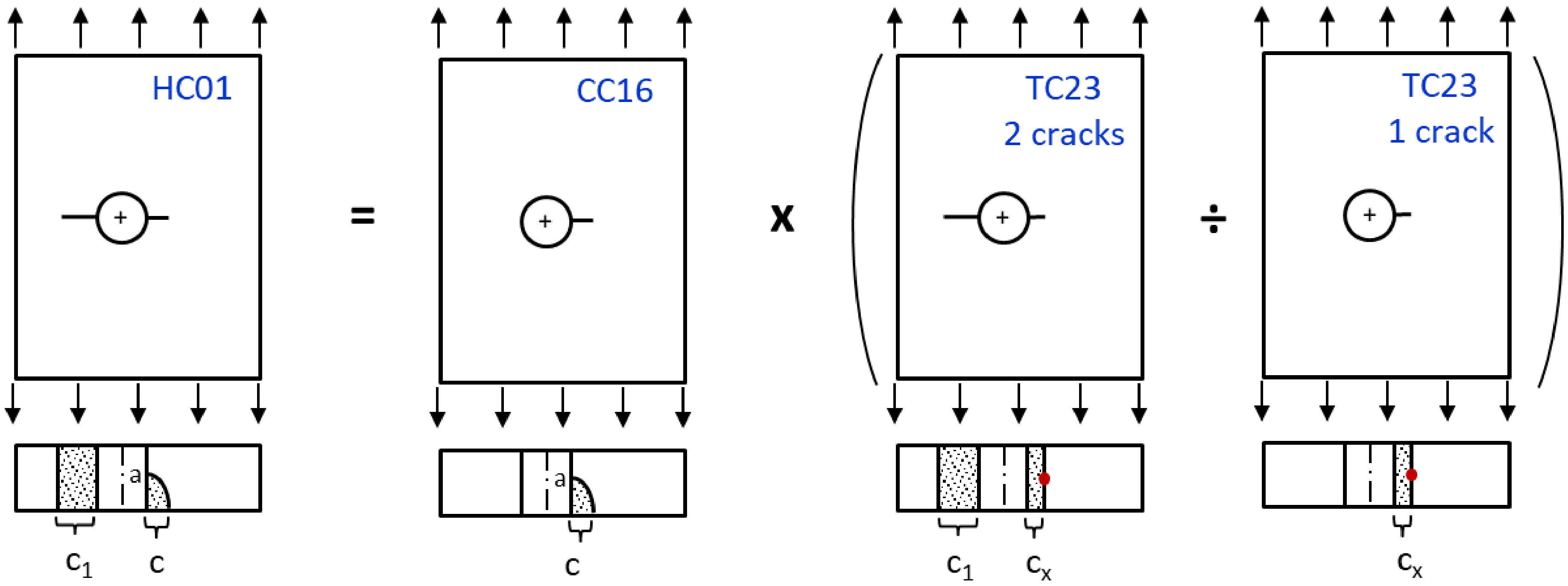
$$K_i^A = \beta_i^A \cdot S_0 \cdot \sqrt{(\pi c_i)}$$

$$K_i^B = \beta_i^B \cdot S_2 \cdot \sqrt{(\pi c_i)}$$

$$K_i^C = \beta_i^C \cdot S_3' \cdot \sqrt{(\pi c_i)} = \beta_i^C \cdot (S_3 D/W) \cdot \sqrt{(\pi c_i)}$$

What Happens (First) After Corner Cracks Transition?

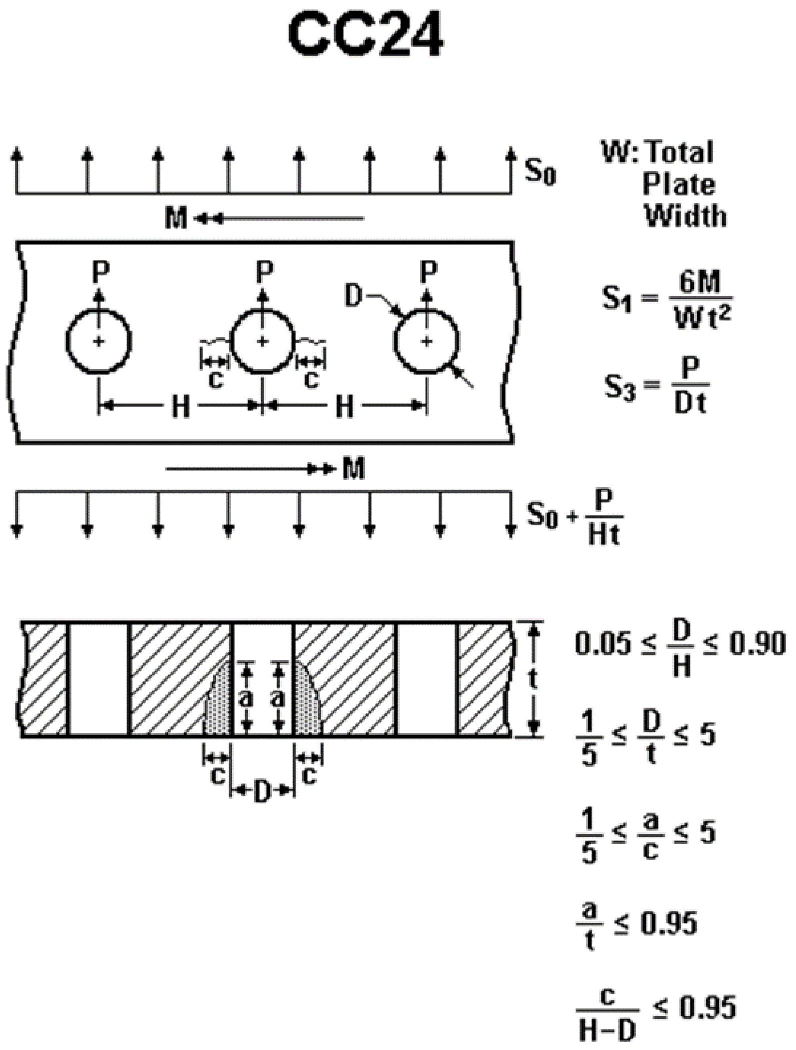
- Compounding approach for "hybrid crack" solution HC01 with corner crack and through crack on opposite sides of hole (Guo, c2013-14)



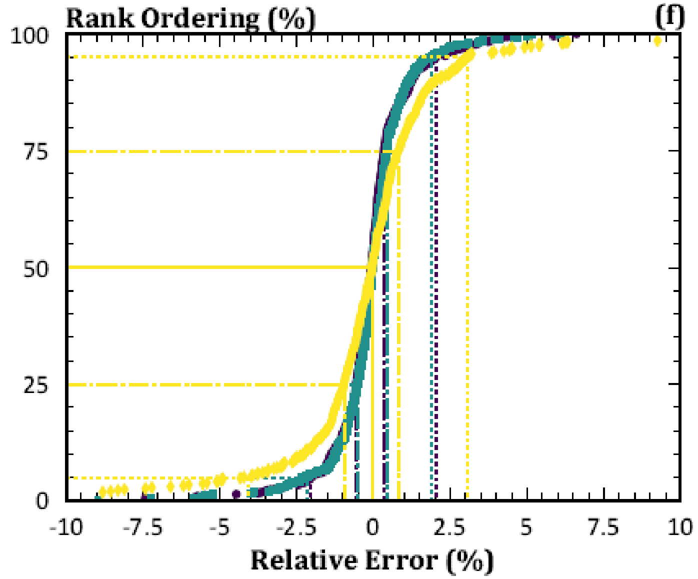
c_x is the characteristic crack length

What About Corner Cracks at a Row of Holes?

- Brute force data-table approaches are impractical for one/two corner cracks at one hole or two cracks at every hole in a row of holes (too many DOFs)
- Alternative: GP model trained with results from 500 geometries semi-randomly selected using LHS (Sobotka and Ismonov, c2020ff)

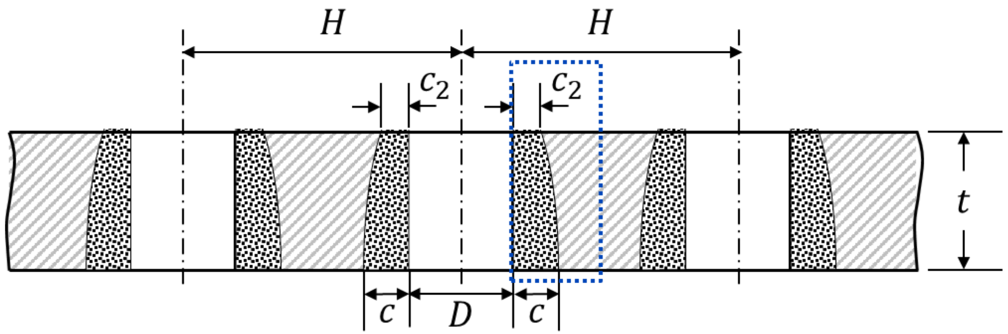


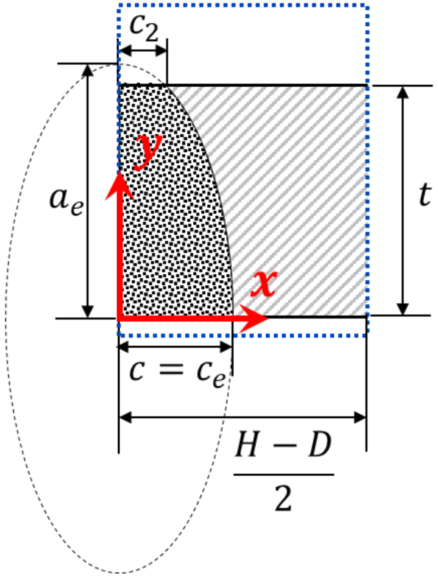
Representative verification: c-tip, 2 cracks at 1 hole, three loading modes

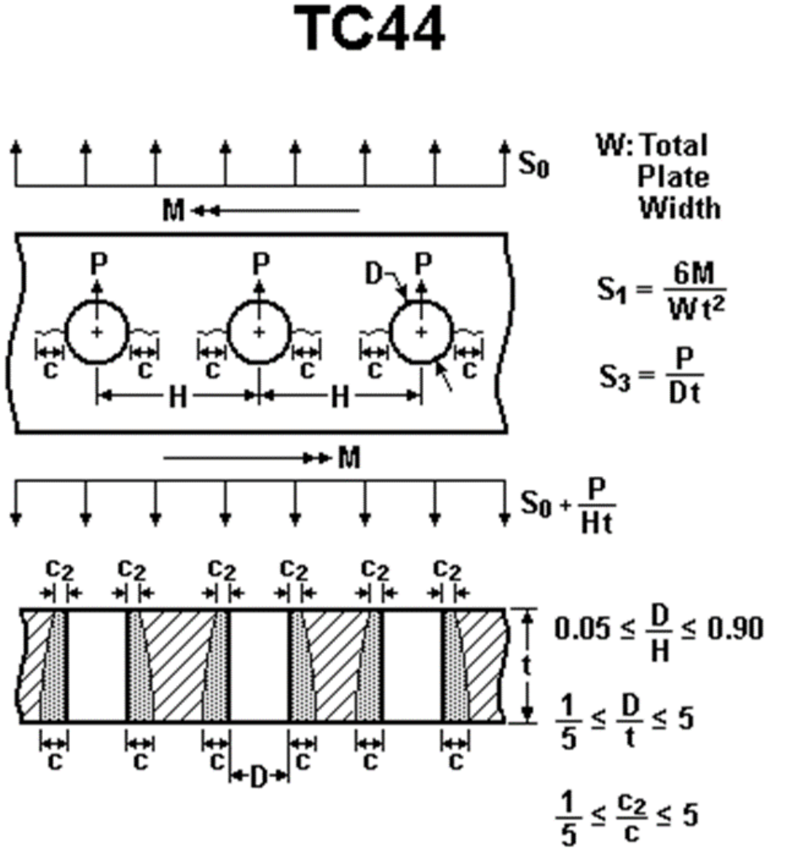


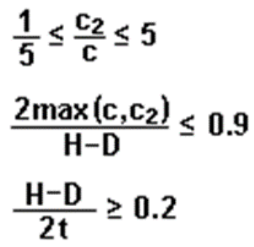
What Happens After Transition?

- Corner cracks at a row of holes will transition to through cracks at a row of holes
- The through cracks will (in general) not be straight
  - Gradual shape change after transition
  - Never straight if out-of-plane bending occurs
- Approach: GP model for curved through cracks



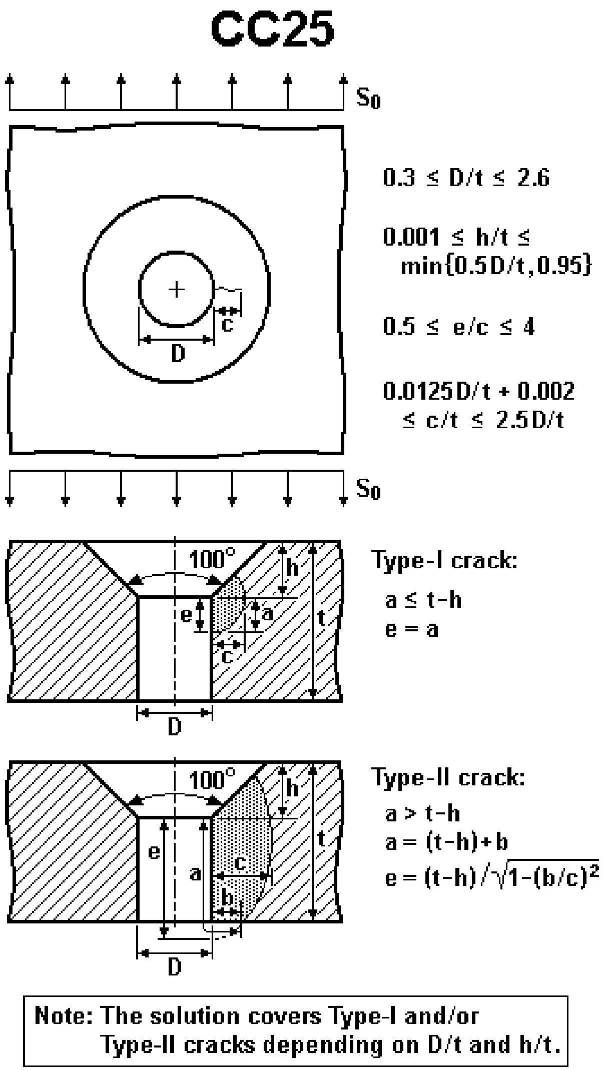




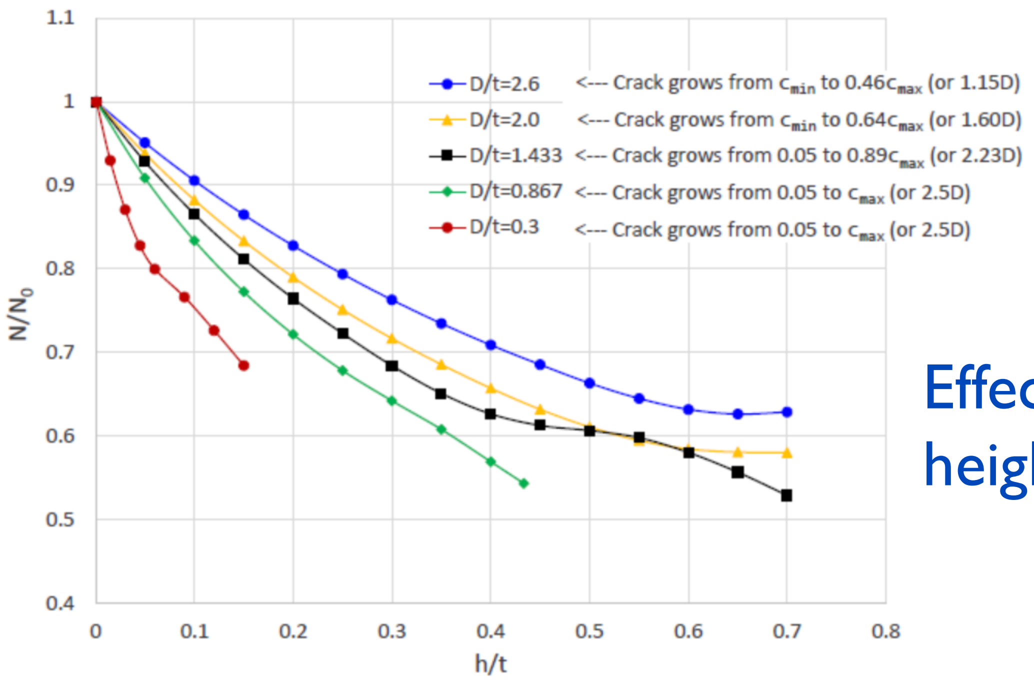


Development by Sobotka and Ismonov (c2022ff)

What about Countersunk Holes?



- Based on the Cronenberger (2011) solution
- Covers type-I and/or type-II cracks depending on a/t
- Infinite plate with countersunk angle of $100°$
- Loading by remote tension (S_0) only

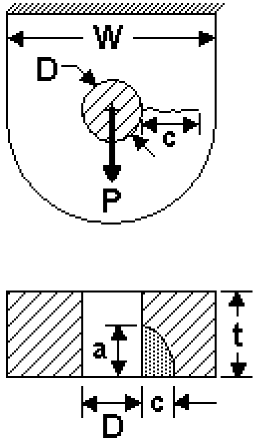


Effects of countersink height on fatigue life

What if the Hole is in a Lug?

- WF approach for pin-loaded holes in plates can be applied to lugs (Sobotka, c2015)
- Determine crack-plane stresses for uncracked lug and use WF for crack in plate
- Verify WF SIF through direct comparisons with FE analyses of cracked lugs

CC19



$S_3 = P/Dt$

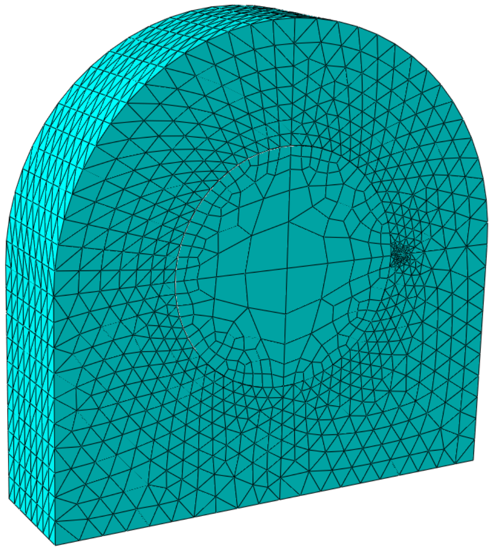
$0.1 \le D/2t \le 10$

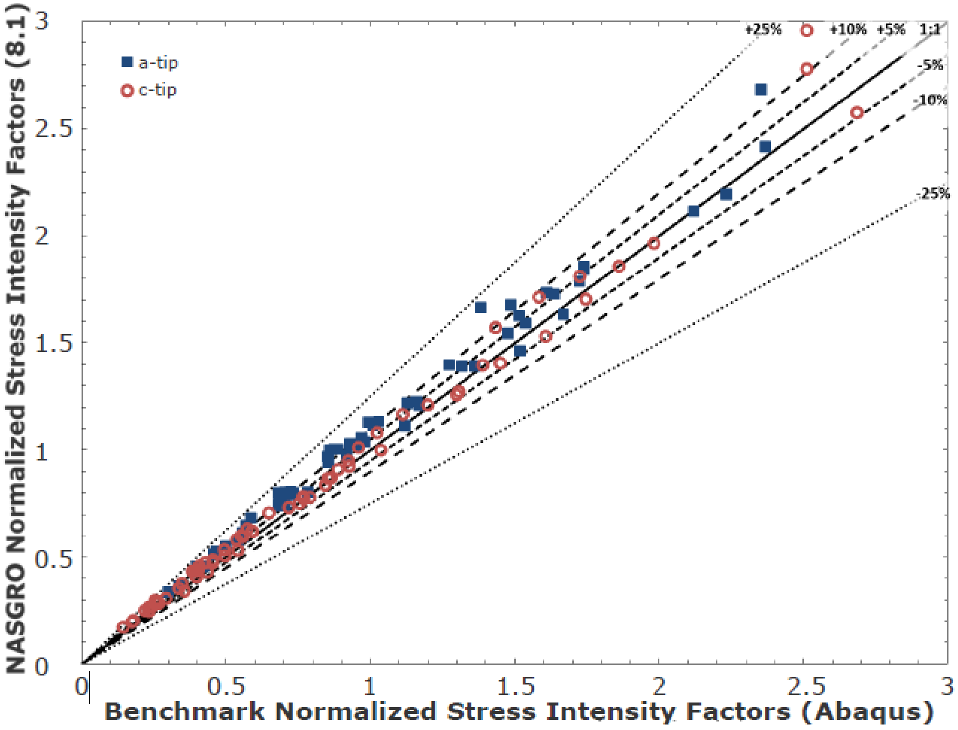
$1.25 \le W/D \le 10$

$0 \le c/(W-D) \le 0.45$

$a/c \ge 0.1$

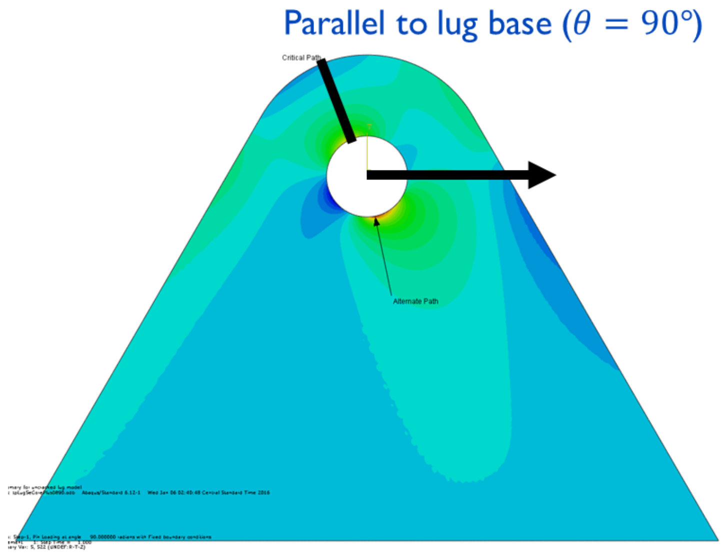
$0 \le a/t \le 0.95$

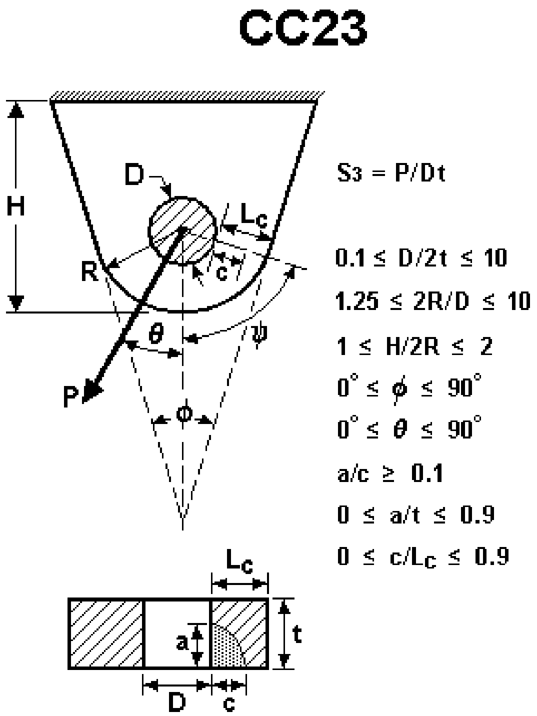


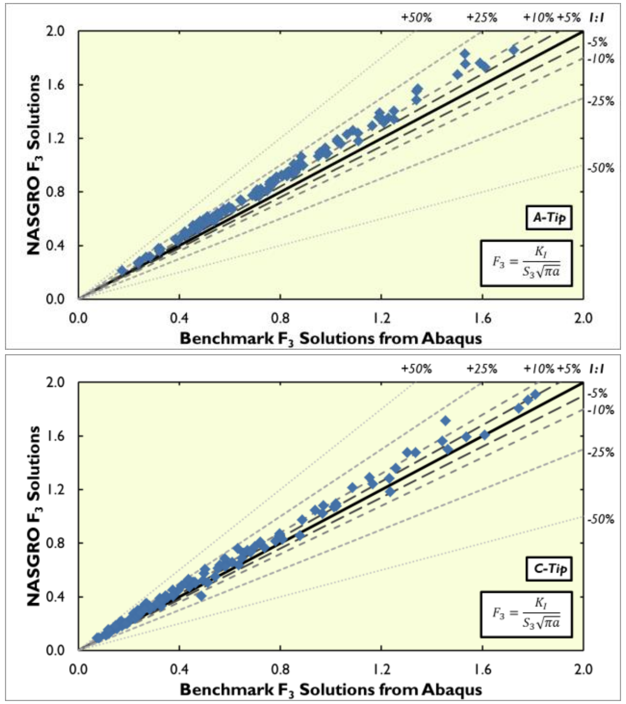


What to do about the Variety of Lug Geometries?

- WF approach is especially powerful for complicated problems like tapered lugs
- Use FE to get crack-plane stresses for wide range of lug tapers and loading angles
  - Find most likely crack plane (maximum opening stress angle at hole in uncracked body)
  - Crack can be on short ligament or long ligament
- Combine crack-plane stresses with WF for corresponding crack in pin-loaded plate
- Sobotka: ASIP, 2016; ICAF, 2019

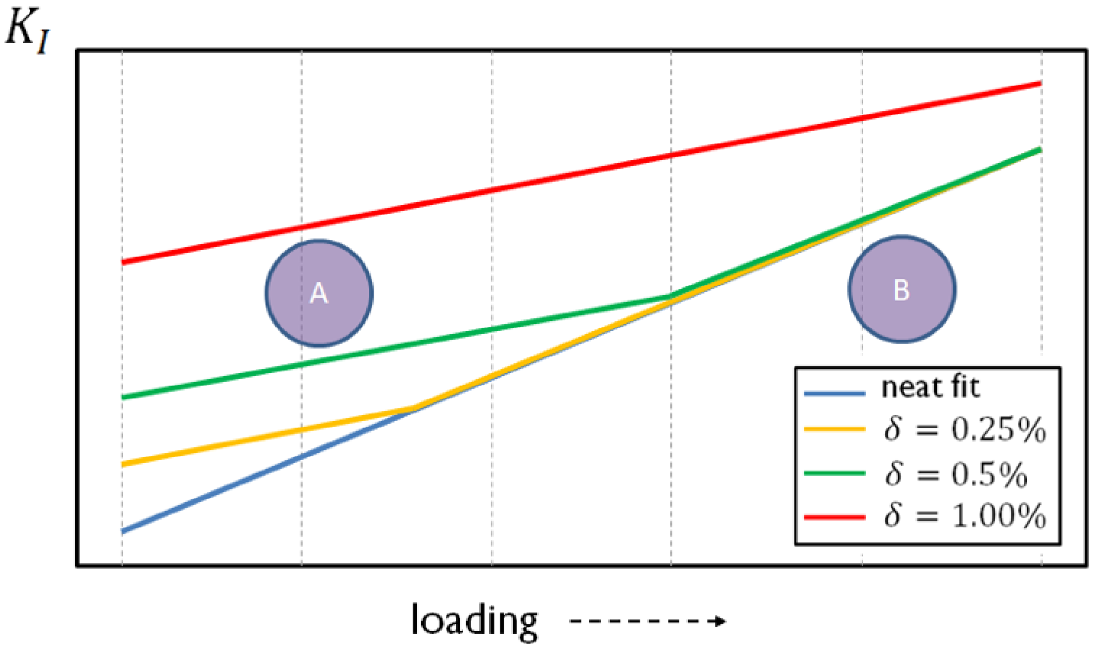






What if the Hole Has an Interference-Fit Fastener?

- Through crack at hole with interference-fit fastener (Haikal and Sobotka, ASIP, 2021)
- Interference causes a change in contact conditions as loading increases – two regimes
  - A. Active interference increases SIF but decreases rate of increase of SIF with load
  - B. Neat-fit solution (no interference effects)
- Interference residual stress approach does not work



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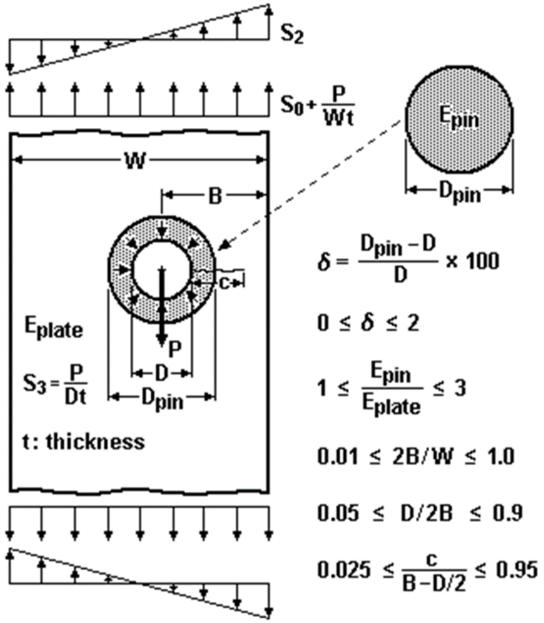
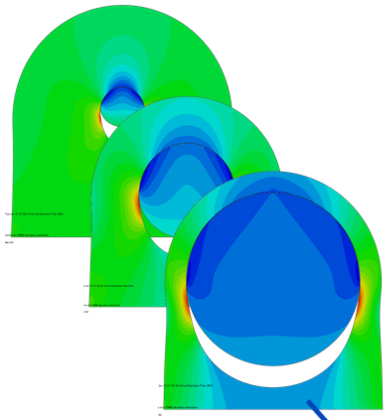


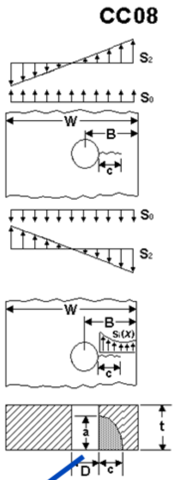
A Few Words about SIF Verification

- Engineering SIF solutions should be verified with independent benchmark solutions
- Evolving paradigm to do this routinely (Sobotka and McClung, 2019)
  - Large numbers (scripting) of high-quality FE SIF calculations (Abaqus or StressCheck)
  - Wide solution space interrogated efficiently with LHS

Stress Solutions

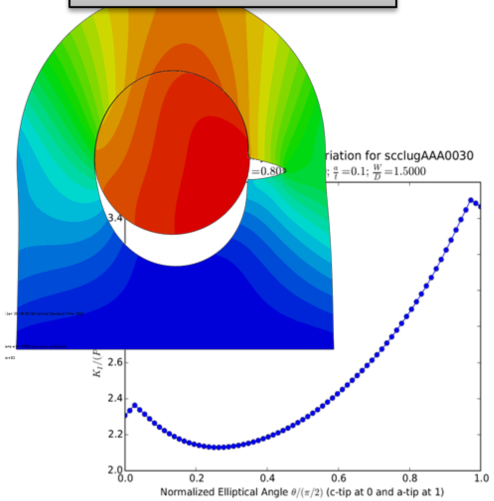


Weight Function

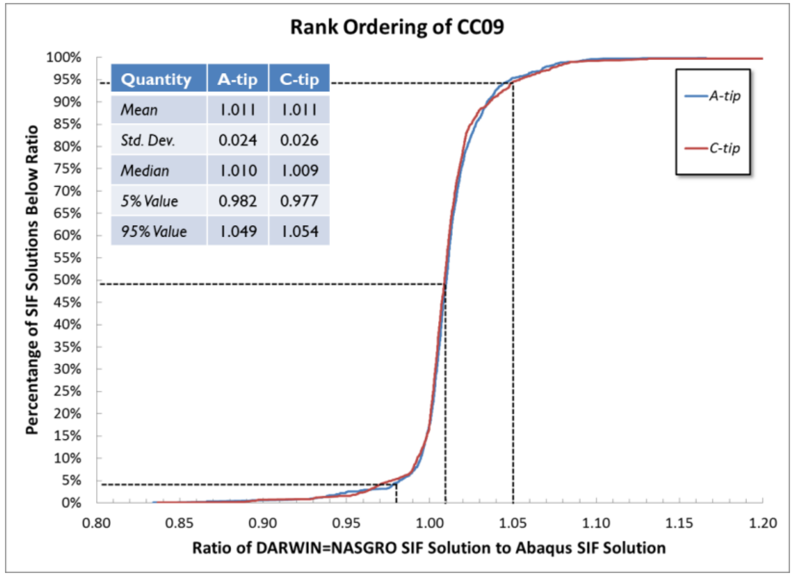


$$K_I = \int_{-c}^{+c} S(x) \times \mathbf{w}(\mathbf{x})dx$$

Abaqus K Solutions



FEA → Domain Integral → K_I



New Tools for New Challenges

- Response surfaces built with Gaussian Process (GP) models
  - As the number of DOFs in a model increases, so does the computational cost
  - GP models require fewer calibration points to achieve the same level of accuracy as conventional spline functions
- Principal Component Analysis (PCA) for large datasets
  - Method to determine orthogonal modes ordered to maximum variability of function
  - Enables reduced-order approximations with high accuracy and minimal data storage
- Automation using scripting capabilities
  - Scripting capabilities in Abaqus, StressCheck, and other tools enable high-fidelity models to be built, executed, and post-processed using internal CAE routines
  - Enables more and better analyses to be performed during development and verification

Future Work

- Additional loading modes, including out-of-plane bending
  - Bivariant effects
  - Curved through cracks
- Additional geometric factors
  - Offset holes in lugs
  - Countersunk and recessed holes
- Cold-expanded holes
- Multiple holes and multiple cracks
- Filled vs. open hole solutions
- Broad array of lug geometries
- Interference-fit and clearance-fit holes
- Multi-site damage

Summary

- Legacy data-table solutions for corner cracks at holes in wide plates under uniform remote loading provide a solid foundation for engineering applications
- But regular data tables alone are inadequate for practical applications
  - Finite geometries with many degrees of freedom
  - Complicated stress distributions
- Advanced techniques developed to address the challenges
  - Sophisticated compounding methods
  - Univariant and bivariant weight function methods
  - Gaussian Process models
  - Rigorous verification protocols
- Accurate, fast, and robust SIF solutions are now available in the NASGRO® engineering software to support damage tolerance design and analysis
- Future work will continue to extend the solution space into new configurations

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